Assignment Markets: Theory and Experiments[⋆]

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Abstract

We experimentally test convergence to the core in two-sided markets for heterogeneous indivisible goods under different trading institutions. We use bargaining and strategic games as predictors that naturally generalize the core, accommodating non-equilibrium behavior. The performance of the competing theories reflects the differences in trading procedures—market outcomes are close to Nash equilibrium predictions under auction-like institutions and close to bargaining for institutions that feature decentralized negotiations. This difference may be driving the documented effect of fewer no-trade outcomes at the expense of a higher chance of suboptimal match under free-form bargaining.

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1. Introduction

The assignment game represents markets characterized by indivisibility and heterogeneity of goods and inflexibility of supply and demand satiated at exactly one unit. In a classic contribution, Shapley and Shubik (1971) proved that core allocations solve the optimal assignment problem and correspond one-to-one to competitive equilibrium allocations. Despite this result appearing a half-century ago and a growing body of recent research studying if decentralized individual behavior can reach core allocations[1], there is little evidence or explanation for when or how it fails and inefficient outcomes occur. In this paper we contrast the predictions of several non-cooperative generalizations of the core with the behavior of participants in laboratory experiments under different institutions[2]. Our models generalize the core, and admit non-core allocations observed in experiments. These non-core allocations are predictions of "what can go wrong" in assignment markets when trading is implemented in practice.

Our strategic model is a market game adapted from Dubey (1982), Simon (1984), and Benassy (1986). Buyers and sellers are allowed to bid simultaneously, with a clearing house picking an allocation consistent with the bid and ask prices submitted respectively by buyers and sellers. We show that every Nash equilibrium allocation is a competitive equilibrium allocation of an economy composed exclusively of the traded goods. That is, inefficiencies may arise due to coordination failures—buyers and sellers may fail to trade some goods that would be sold in an optimal assignment. In an alternative noncooperative game, following Pérez-Castrillo and Sotomayor (2017), we allow sellers to post prices before buyers are allowed to bid. Subgame perfection in this, now sequential, game selects the best competitive equilibrium for sellers and, therefore, an optimal assignment.

Our bargaining model is a version of an asymmetric Nash bargaining solution that we call Pairwise Nash Core (PN-Core). The PN-Core consists of allocations that maximize the asymmetric Nash product in every matched pair subject to incentive compatibility constraints. These allocations may be suboptimal in the utilitarian sense and inconsistent with competitive prices. By contrast to the strategic models that capture miscoordination, this model captures the subjects' desire for fairness – the bargaining weights represent the share of the surplus that each subject considers warranted.

The possibility of bargaining simultaneously with multiple partners and reneging on agreements, the presence of search frictions as well as the choice of a structured trading or a bargaining protocol has been suggested before as a possible determinant of "who matches with whom" in such markets, especially in the experiments of Nalbantian and Schotter (1995) and Agranov and Elliott (2021). Based in part on this

[1] A summary of results for assignment problems is offered by Roth and Sotomayor (1990, Chapter 8) and more recently by Núñez and Rafels (2015).

[2] All code necessary to replicate the simulations and data analysis can be found in a Mendeley Data repository at `https://data.mendeley.com/datasets/nj7w7g8nft/1` (Dolgopolov, 2023)

evidence, we construct experimental treatments that should highlight the bargaining and non-cooperative sides of our assignment markets: some treatments are structured auctions, and the other follow a free-form negotiation protocol with reneging.

In the first treatment, we adapt a double auction as introduced by Smith (1962) and now common in market experiments with homogeneous goods (see the survey by Friedman (2018)). In the second treatment, we allow sellers to post prices before buyers bid for their goods. In the third treatment, we allow for bilateral, private, free format communication between buyers and sellers, mimicking real-estate interactions, with the possibility of striking deals that traders can renege on by making another deal. In these three treatments, we do not inform traders of the valuations of other traders, and we repeat the experiment for several rounds with fixed roles and valuations to allow for learning of market-relevant information and for experience. The fourth treatment offers a robustness check under full information about valuations.

We find that bargaining models are a better fit for treatments with private bilateral negotiations, while competitive prices and Nash equilibrium assignments occur more often in the structured markets. As in other experiments,[3] communication favors coordination, leading to the optimal assignment, but also introduces either bargaining or pro-social behavior, in our case, exemplified by the frequency of suboptimal matchings. Finally, the treatment where sellers move first favors them and allows them to reap some gains from the ability to commit. However, in spite of being induced by the unique subgame-perfect equilibrium, the optimal assignment is not reached in the posted price treatment as frequently as in the treatment with communication. Nalbantian and Schotter (1995) document the same effects with more matches overall in the free-form negotiation treatment, but a larger proportion of suboptimal matches compared to an English auction. We explain these effects through the difference between bargaining and Nash equilibria.

The remainder of the paper is organized as follows. In Section 2 we first define the assignment game, the core and the competitive equilibrium. Next, we develop the non-cooperative counterpart, the strategic market game, and characterize the Nash equilibria through their connection to the competitive equilibria. We then do the same with the bargaining model. In Section 3, we describe the experimental design. We first describe the experimental market and use it to illustrate the predictions of the theoretical models. We then describe the four treatments. In Section 4, we relate theoretical predictions to experimental results and conduct several checks for alternative explanations. Section 5 concludes with gaps and avenues for future research.

[3]See the survey in Martinelli and Palfrey (2020).

2. Theoretical Predictions

2.1. The assignment game

Let B and S denote respectively the (disjoint) sets of buyers and sellers, with $|B| = M$ and $|S| = N$. Each seller has one good to sell, and we, therefore, use S for the set of goods as well. Each buyer j has a valuation for each seller's good i, denoted h_{ij}, while each seller has a reservation value for her good c_i. Possible market operations are transfers of goods from a seller to a buyer and transfer of money within a pair of trading partners.

An *assignment* of buyers to sellers can be represented by an $N \times M$ binary matrix x with $x_{ij} = 1$ whenever good i is allocated to buyer j, and $x_{ij} = 0$ for all j if the good i is not sold. The set of assignments is then:

$$X = \{(x_{11}, \ldots, x_{NM}) : x_{ij} \in \{0, 1\}, \sum_{j \in B} x_{ij} \leq 1 \text{ for all } i \in S\}.$$

We will also use the matching function $m : (S \cup B) \to (\emptyset \cup S \cup B)$ for convenience with the interpretation that $m(i) = j$ means that the seller (good) i is assigned to buyer j and vice versa for $m(j)$, $j \in B$.

Payoffs are linear in money. If some buyer buys a good from seller i, she has to pay p_i. Every buyer is only interested in buying one good, and her payoff is $\max_{i \in S}(x_{ij} h_{ij}) - \sum_{i \in S} p^B_{ij} x_{ij}$. The seller obtains payoff $p_i - c_i$. Let $a_{ij} \equiv \max(0, h_{ij} - c_i)$ be called the *trade surplus* of the buyer j and seller i. These surpluses form the $N \times M$ matrix A. We will assume that the values and costs are fixed and refer to (S, B) as an *economy* consisting of sellers S and buyers B.

Following Aumann (1961), we define the core in terms of dominance and also use this approach later for our bargaining concept. Consider the set of valid trading pairs $S \times B = \{(i, j) : i \in S, j \in B\}$, called the *coalition structure*. Then a (set-valued) *characteristic function* assigns a set of pairs of payoffs to every coalition. The characteristic function v is defined as $v(i, j) = \{(u^S_i, u^B_j) \in \mathbb{R}^2_+ : u^S_i + u^B_j \leq a_{ij}\}$. It is the set of all payoffs attainable by the coalition.

The *feasible set* $U \subset \mathbb{R}^{M+N}_+$ is the set of payoff vectors or *imputations* that are attainable in some assignment. In particular, an $(M+N)$-payoff vector $u = (u^S_1, \ldots, u^S_N, u^B_1, \ldots, u^B_M)$ is in the feasible set U if and only if there is an assignment $x \in X$ such that $u^S_i + u^B_j = h_{ij} - c_i$ if $x_{ij} = 1$, $u^S_i = 0$ if $\sum_{j \in B} x_{ij} = 0$, and $u^B_j = 0$ if $\sum_{i \in S} x_{ij} = 0$.

A payoff vector $\hat{u} \in \mathbb{R}^{N+M}_+$ is said to *dominate* a payoff vector $u \in \mathbb{R}^{N+M}_+$ with respect to characteristic function v if for some pair $(i, j) \in S \times B$, there is $(\hat{u}^S_i, \hat{u}^B_j) \in v(i, j)$, s.t. $u^S_i < \hat{u}^S_i$ and $u^B_j < \hat{u}^B_j$. A *core imputation* is a feasible payoff vector that is not dominated by any other feasible vector with respect to characteristic function v.

The optimal assignment that maximizes the total payoff of all players is obtained by solving the assignment problem:

$$\max_{x \in X} \sum_{i \in S} \sum_{j \in B} x_{ij} a_{ij} \text{ such that } \sum_{i \in S} x_{ij} \leq 1 \text{ for all } j \in B. \quad (1)$$

Shapley and Shubik (1971) show that the set of core imputations are solutions to the dual program:

$$\begin{gathered}\min \textstyle\sum_{i\in S} u_i^S + \sum_{j\in B} u_j^B, \text{ such that} \\ u_i^S + u_j^B \geq a_{ij} \text{ for all } (i,j) \in S \times B, \text{ and } u_i^S \geq 0, u_j^B \geq 0.\end{gathered} \tag{2}$$

To describe competitive behavior, let $Y_j^B = \{0,1\}^N$, with typical element $y_j^B = (y_{ij}^B)_{i\in S}$, represent the set of possible demand vectors for buyer $j \in B$, with the interpretation $y_{ij}^B = 1$ if buyer j demands good i and $y_{ij}^B = 0$ otherwise. That is, a buyer can acquire one good, several, or none. Similarly, let $Y_i^S = \{0,1\}$, with typical element y_i^S, be the set of possible supply decisions by seller i, with the interpretation that $y_i^S = 1$ if i sells her good and $y_i^S = 0$ otherwise.

A *competitive equilibrium* for the economy (S, B) is a pair (y, p), $y = ((y_i^S)_{i\in S}, (y_j^B)_{j\in B}) \in \prod_{i\in S} Y_i^S \times \prod_{j\in B} Y_j^B$ and $p = (p_i)_{i\in S} \in \mathbb{R}_+^N$ such that

$$\begin{gathered}\max_{i\in S}(y_{ij}^B h_{ij}) - \textstyle\sum_{i\in S} p_i y_{ij}^B \geq \max_{i\in S}(y_{ij}'^B h_{ij}) - \sum_{i\in S} p_i y_{ij}'^B \text{ for all } y_j'^B \in Y_j^B, \text{ for all } j \in B, \\ (p_i - c_i) y_i^S \geq (p_i - c_i) y_i'^S \text{ for all } y_i'^S \in Y_i^S, \text{ for all } i \in S, \text{ and} \\ \textstyle\sum_{j\in B} y_{ij}^B = y_i^S \text{ for all } i \in S.\end{gathered} \tag{3}$$

The first set of conditions represents utility maximization by buyers, and encodes the assumption that buyers can enjoy at most one good. The second set of conditions represents profit maximization by sellers. The third set of conditions are market clearing conditions for each of the goods. Although, in the core the buyers can only match with one seller and buy at most one good, in the competitive equilibrium they could, in principle, buy several goods. However, since they only enjoy one of these goods, the set of core imputations coincides with the set of competitive equilibrium imputations.

2.2. Strategic market game

We can now define the strategic solution. In the strategic *market game* $\Gamma(S, B)$ each seller $i \in S$ submits a price $p_i^S \in \mathbb{R}_+$, and each buyer submits an N-vector of positive bids $p_j^B \in \mathbb{R}_+^M, p_j^B = (p_{1j}^B, ..., p_{Mj}^B)$. The set of admissible prices/bids for player $k \in B \cup S$ is denoted W_k. An *offer profile* combines actions of all players in a tuple $w = (p_1^B, \ldots, p_M^B, p_1^S, \ldots, p_N^S) \in W = \prod_{k\in B\cup S} W_k$.

Once all bids and prices are submitted, a clearing house chooses an assignment or a lottery over assignments from the set X to maximize surplus

$$\pi(x, w) = \textstyle\sum_{i\in S} \sum_{j\in B} x_{ij}(p_{ij}^B - p_i^S),$$

A clearing house draws from the following set of surplus-maximizing assignments:

$$\bar{X}(w) = \{x \in X : \pi(x, w) \geq \pi(x', w) \text{ for all } x' \in X\}.$$

To ensure that the clearing house prefers more trade even when arbitrage is zero, we assume that it chooses assignments that are not ray-dominated (Simon, 1984):

$$F(w) = \{x \in \bar{X}(w) : \text{there is no } x' \in \bar{X}(w) \text{ such that } x' \neq x \text{ and } x'_{ij} \geq x_{ij} \text{ for all } i \in S, j \in B\}.$$

We also assume that the clearing house randomizes over the whole $F(w)$ with full support.

Once the clearing house chooses an assignment $x \in F(w)$, the market clears at the buyers' prices,[4] and buyers and sellers get the respective payoffs

$$\max_{i \in S}(x_{ij}h_{ij}) - \textstyle\sum_{i \in S} p^B_{ij}x_{ij} \quad \text{and} \quad \textstyle\sum_{j \in B}(p^B_{ij} - c_i)x_{ij}.$$

Before we describe the set of Nash equilibria of the market game, note that for every subset $S' \subseteq S$ of sellers (including the empty set), we can define a smaller economy (S', B) with S' as the set of sellers and B as the set of buyers. With a slight abuse of notation, for any $y' \in \prod_{i \in S'} Y_i \times \prod_{j \in B} Y_j$, define the allocation $x(y') \in X$ as

$$x(y') = (x \in X : x_{ij} = y_{ij} \text{ if } i \in S' \text{ and } j \in B, \text{ and } x_{i'j} = 0 \text{ if } i' \notin S' \text{ and } j \in B).$$

If $x(y') \in F(w)$ for some bid profile $w = (p^B_1, \ldots, p^B_M, p^S_1, \ldots, p^S_N)$ such that

$$y'_{i'j} = 1 \quad \Rightarrow \quad p^S_i = p^B_{ij} \quad \text{for every } i' \in S' \text{ and } j \in B,$$

then $F(w) = \{x(y')\}$, and w is a Nash equilibrium. Moreover, for every $i' \in S'$ and $j \in B$ such that $y'_{i'j} = 1$, we have $p^S_i = p^B_{ij} = p'_i$. In other words, w induces the same assignment and the same prices for the goods in S', while the goods in $S \setminus S'$ are not sold. We will say in this case that a profile w induces the same allocation as the competitive equilibrium (y', p').

The next two theorems characterize the Nash equilibria of this game and relate them to competitive equilibria. The proofs are collected in Appendix A.

Theorem 2.1. *If (y', p') is a competitive equilibrium for economy (S', B) for some $S' \subseteq S$, then there is a Nash equilibrium bid profile w that induces the same allocation.*

An immediate corollary of Theorem 2.1 is that every competitive equilibrium allocation of the original economy (S, B) can be supported by a Nash equilibrium. However, other allocations can be supported as well; in fact, recalling that in our environment every good represents a different market, any arbitrary subset of markets can be shut down in a Nash equilibrium. This is the result of a coordination failure. Intuitively, markets for particular goods may not open because each side of the market expects the other side not to show

[4] Allocating surplus to one side of the market follows the convention in Dubey (1982) and also reflects the experimental treatment where one side of the market has the full market power.

up. Such allocations are the semi-Walrasian allocations, which, as shown in Mas-Colell (1982), are more difficult to destabilize than other non-equilibrium allocations. Another view on semi-Walrasian equilibria is given by "blocking" in Mas-Colell (1982). Adapting it to our non-cooperative setup, we say that a profile of actions a is k-blocked if there is a set of k-players that can all strictly improve their payoffs by jointly changing their actions. Mas-Colell (1982) defines k-blocked allocations as those for which the smallest set of players necessary to change the market outcome is of size k. We ignore the arbitrage considerations from the original definition as they are irrelevant in the assignment market.

The bipartite nature of the assignment market simplifies the analysis of these semi-Walrasian equilibria compared to Mas-Colell (1982). All non-core allocations are either 1-blocked or 2-blocked. Two players at the most need to change actions simultaneously to destabilize any Nash equilibrium by making a market active or inactive. Therefore semi-Walrasian allocations that are not Warlasian equilibria are exactly the 2-blocked allocations. If the market is out of semi-Walrasian equilibrium, at most one player is needed to change the outcome, so the remaining, non-equilibrium allocations are 1-blocked. Even though at most 2-blocking allocations need to be considered, two players fixing their coordination problem may affect many or all of the remaining agents. For example, while we only need to change actions by the third buyer or the third seller in the assignment [*210*] to nudge the markets toward the competitive equilibrium, the matching among active traders changes as well—buyer 1 is rematched with seller 3 instead of seller 2.

A converse result to theorem 2.1 also holds.

Theorem 2.2. *If $F(w) = \{x\}$ and w is a Nash equilibrium of a market game $\Gamma(S, B)$, then there is a competitive equilibrium (y', p') for some economy (S', B), where $S' \subseteq S$, such that x induces the same allocation as (y', p').*

The following corollary follows. Intuitively, if all markets open, the strategic game leads to a competitive equilibrium for the complete economy.

Corollary 1. *If w is a Nash equilibrium of $\Gamma(S, B)$ such that $F(w) = \{x\}$ satisfying $\sum_{j \in B} x_{ij} = 1$ for all $i \in S$, then x is a solution to (1).*

As an alternative noncooperative game, inspired by Pérez-Castrillo and Sotomayor (2002, 2017), consider a sequential game with complete information in which sellers choose their prices first before buyers choose their bids, with the clearing house choosing the final allocation as before. By standard arguments, every Nash equilibrium of the simultaneous game corresponds to a Nash equilibrium of the sequential game in which the buyers choose the same bid no matter what happens in the first stage of the game. Interestingly, following Pérez-Castrillo and Sotomayor, there is a unique subgame-perfect equilibrium path, and it leads to the best allocation for sellers in the core.

2.3. Bargaining

The other class of models that we consider are bargaining models, which are candidates for predicting behavior in treatments with communication.

We will first define the asymmetric Nash bargaining solution (NBS) for weights α, and sets of sellers and buyers S and B, denoted $b(\alpha, S, B)$ to be the set of solutions of the following optimization problem:

$$\max_{u \in U} \prod_{i \in S} (u_i^S)^{\alpha_i} \prod_{j \in B} (u_j^B)^{\alpha_j}, \tag{4}$$

where $u = (u_1^S, \ldots, u_N^S, u_1^B, \ldots, u_M^B)$, and $\alpha \in \triangle_{M+N}$, the $(M+N)$-simplex, $\alpha_i, \alpha_j > 0$[5] and $\sum_{i \in S} \alpha_i + \sum_{j \in B} \alpha_j = 1$.

For a single seller in S and a single buyer in B, this is a convex bilateral bargaining problem because the set U is convex, but this is not generally true. Every outcome of asymmetric Nash bargaining is Pareto efficient, but there is a problem with using this as a solution concept for the non-convex assignment economy. The classic axiomatic Nash bargaining solution is not well-defined for nonconvex sets, and there are several competing extensions. Depending on the choice among these extensions, the set of asymmetric Nash bargaining solutions is either a subset of Pareto optima or equals it.[6] The details can be found in Appendix C, but in all cases, under free weights α, bargaining solutions offer only a weak prediction. There is also an implicit assumption that all players bargain jointly over all outcomes. We are interested instead in a bargaining model that can both be applied to non-convex feasible sets and capture the pairwise nature of bargaining in an assignment market.

To achieve this, we adapt the Nash bargaining and the core to describe players who negotiate with a fair proportion in mind. In practical terms, this is a stable allocation of a game where subjects are free to choose trading partners, but the prices are determined by the asymmetric Nash bargaining solution. It is only a formal way of capturing bargaining for a "fair" share of trade surplus, knowing what the other players consider fair, and having an option to look for a better deal.

Formally, the Pairwise Nash characteristic function v_α^{PN} for an assignment economy (S, B) with the surplus matrix A and strictly positive weight vector α is a function that assigns to each pair $(i, j) \in S \times B$ a pair of payoffs given by the unique solution to the pairwise Nash bargaining solution for i and j:

$$v_\alpha^{\text{PN}}(i, j) = (x_i, x_j) = b(\alpha, i, j) = \underset{(u_i^S, u_j^B): u_i^S + u_j^B \leq a_{ij}}{\operatorname{argmax}} (u_i^S)^{\alpha_i} (u_j^B)^{\alpha_j}.$$

The value v_α^{PN} is completely independent of other players, unlike the similar definitions of α-effectiveness (Aumann, 1961) and the Nash-effectiveness (Okada, 2010).

[5] Both definitions are common—only strictly positive weights (Miyakawa, 2008), or non-negative weights (Binmore et al., 1986) depending on the chosen axioms. Strictly positive weights ensure that all NBS are Pareto optimal.

[6] The problem of approximating a non-convex Pareto frontier with a set of optimization problems, including the weighted Nash bargaining solution, is frequent in the field of multi-objective optimization (Braun, 2018; Miettinen, 2012).

An imputation u is said to be in the Pairwise Nash (PN-)core of an assignment market A for weights $\alpha \in \triangle_{M+N}, \alpha \gg 0$ if (i) it is feasible, (ii) the payoffs are the outcome of Nash bargaining solution with weights α in each matched pair, and (iii) u is in the core of A with respect to characteristic function v_α^{PN}. Note that this is a non-transferable utility environment.

3. Experiments

3.1. Experimental Economy and Predictions

We will illustrate all predictions with an example borrowed from Shapley and Shubik (1971), which is described in Table 1. This is also the economy used in all experimental treatments.

Table 1: Widget values for buyers and sellers

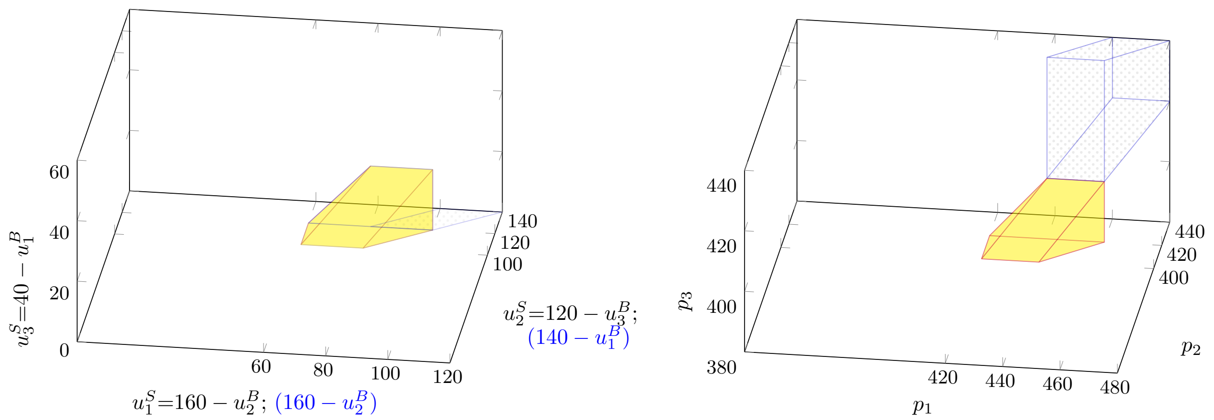
Widgets	Seller's reservation value (E$)	Buyers' valuations (E$)		
(i)	(c_i)	(h_{i1})	(h_{i2})	(h_{i3})
1	360	460	520	400
2	300	440	480	420
3	380	420	440	340

Every row in the table is a seller, and every column is a buyer. For this example, the values a_{ij} comprise the following matrix A, where each element is the joint maximal payoff of buyer j and seller i in experimental dollars (E$):

$$\text{A=} \ (\text{sellers}) \overset{\text{(buyers)}}{\begin{bmatrix} 100 & \textcircled{160} & 40 \\ 140 & 180 & \textcircled{120} \\ \textcircled{40} & 60 & 0 \end{bmatrix}}.$$

The unique optimal assignment is obtained by solving (1), and is given by $x_{12} = x_{23} = x_{31} = 1$ and $x_{ij} = 0$ otherwise.[7] Optimal matches are shown circled in matrix A. A shorter notation will be used below, denoting assignments by three digit numbers. Each digit is the number of the good the buyer bought, e.g. the optimal assignment in the example is [*312*], where buyer 1 bought good 3, buyer 2 bought good 1 and buyer 3 bought good 2. If a buyer does not buy anything, we will write zero.

[7]As pointed by Shapley and Shubik (1971), the optimal assignment is "normally" unique, as in the example below, in our experimental treatments and, generally, with probability 1 if the elements of the assignment matrix are drawn independently from a continuous distribution. There could be several optimal assignments in special cases, for instance if several goods are perfect substitutes for buyers.



a) Core imputations in sellers' payoffs space

b) Competitive equilibrium price vectors

Figure 1: Core, Competitive equilibria and Nash equilibria

Fixing the optimal assignment, we can find core imputations from constraints in (2). Since the constraints corresponding to optimal matches are binding, all core imputations satisfy $u_1^S + u_2^B = 160$, $u_2^S + u_3^B = 120$, and $u_3^S + u_1^B = 40$. The pentahedron depicted in Figure 1a shows all core imputations in the u^S space. In particular, the buyer-optimal imputation is $u = (u_1^S, \ldots, u_3^S, u_1^B, \ldots, u_3^B) = (60, 100, 0, 40, 100, 20)$ and the seller-optimal imputation is $u = (100, 120, 20, 20, 60, 0)$. In between these two extremes lies the core of the game.

Shapley and Shubik (1971) show that in any assignment game all core imputations u can be supported in competitive equilibria by prices

$$p_i = \hat{u}_i^S + c_i \text{ for all } i \in S. \tag{5}$$

Conversely, all competitive equilibrium payoffs are core imputations. We, therefore, have a direct one-to-one mapping from core imputations to equilibrium price vectors,[8] as illustrated by a similar pentahedron in Figure 1b.

From Corollary 1, the only complete assignment that can be induced (with probability one) by a Nash equilibrium is the optimal assignment [*312*]. Deleting rows in matrix A and solving the remaining constraints in (2), we get the other assignments that can be supported deterministically by Nash equilibria. These are, in the order of decreasing total payoff of all players: [*210*], [*320*], [*310*], [*020*], [*010*], [*030*], and [*000*]. The

[8] The one-to-one relation described by equation 5 holds for goods that are optimally assigned; for goods not assigned to any buyer, any price $p_i \geq c_i$ is competitive.

assignment [*210*], shown circled in matrix A below,

$$A = \begin{bmatrix} 100 & \textcircled{160} & 40 \\ \textcircled{140} & 180 & 120 \\ 40 & 60 & 0 \end{bmatrix},$$

is the closest to the optimal assignment by total payoff among all suboptimal assignments. This assignment is the second most frequent after the optimal one, in the auction treatments of our experiments, and it can be supported, for instance, by the following Nash equilibrium profile:

$$\begin{aligned} w &= (p_1^S, p_2^S, p_3^S, p_1^B, p_2^B, p_3^B) \\ &= (470, 430, 440, (460, 430, 380), (470, 430, 380), (400, 420, 380)). \end{aligned}$$

The prices in all Nash equilibria with assignment [*210*] are shown as a blue dotted region in Figure 1, the corresponding payoffs of the sellers shown on the axes after the semicolon in blue. The price of the unsold third good has to be above a certain level as shown in Figure 1b, but the utility of seller 3 is zero as shown in Figure 1a. This region, and similar regions for other active sets of market motivate our main experimental hypothesis:

Hypothesis 1. *The players play Nash equilibria, or equivalently semi-Walrasian equilibria.*

If the hypothesis is true, the payoffs would fall into either of the two regions in Figure 1, or possibly other Nash equilibrium regions for other less efficient matchings if they are present. In all cases the matchings should be [*210*], [*320*], [*310*], [*020*], [*010*], [*030*], or [*000*]. The prices should consistute a competitive equilibrium for the active markets. Thus, the Nash equilibria formulate a well-defined set of predictions, both in terms of assignments and in terms of prices.

The core and semi-Walrasian equilibria are institution-free models. Our secondary goal is to see how the institutions affect the tendency of subjects to follow the Nash equilibrium logic. We vary the institutional framework between our four treatments (described in the next section) between a market institution and a free-form negotiation. This setup is meant to test the secondary hypothesis:

Hypothesis 2. *The players play according to the PN-Core instead of Nash equilibria if the market institution is replaced by free-form negotiation.*

The PN-Core is only one of possible bargaining solutions. Most of bargaining solutions, including Nash bargaining, imply Pareto optimality, so we would also expect more Pareto Optimal outcomes for these treatments. By implication, all markets should generally be active. The only exception is that exactly buyer 3 and seller 3 can remain unmatched, because their surplus from trading is zero and such assignment does not violate Pareto Optimality. With this exception, in terms of assignments, bargaining models predict that all goods are traded. At the same time, price predictions of these models are very weak if no restrictions are imposed on the weights. Almost any feasible imputation with a core assignment can be explained by PN-core for some weights. However, it is unlikely that the weights are highly heterogenous or that they significantly change over time: we keep experimental subjects in the same groups, and never vary the payoff matrix, so there is no change in features that should be relevant to bargaining positions. Therefore, instead of only asserting that behavior should be explainable by bargaining models with some weights, we will additionally test whether the weights are stable over time and similar across groups.

Finally, the core outcomes are Pareto optimal and rationalizable by Nash equilibria, Nash bargaining solution, and the PN-Core Appendix E. Apart from corner cases, the core is exactly the set of outcomes that combines all these features. Therefore core outcomes are the intersection of all models. They should be frequent in all treatments regardless of the implemented institution.

Hypothesis 3. *The core outcomes are frequently chosen by subjects in all treatments.*

3.2. Experimental design

Experimental sessions were conducted with 234 GMU undergraduate subjects in total, broken down into groups of six (three buyers and three sellers) for a total of thirty nine groups. Each of the main treatments was run with 10 groups of 6 subjects. Experiments were conducted in oTree (Chen et al., 2016). The three main treatments were held from September, 2018 to April, 2019 in person, and the fourth robustness treatment was conducted online in October–December 2020. The first 5 rounds were discarded as learning rounds and dropped from all statistical analyses, but this was not announced to the subjects. The software matched participants in groups of 3 buyers and 3 sellers with the matching and roles unchanged over the entire experiment. Subjects were recruited for 90 minutes. Earnings were calculated in the experimental currency and converted to U.S. dollars at the end of the experiment at a rate of E$ 5 to US$ 1. Only the earnings from one randomly chosen round were paid, with a mean payment of $13.86 across treatments.

In our experimental treatments, we inform traders of the number of other buyers and sellers in the group, as well as each trader's own parameters (e.g. the vector $(h_{ik})_{i \in S}$ if trader k is a buyer, and c_k if trader k is a seller). As in realistic market conditions, we do not normally inform them about the parameters of other traders, except for the robustness treatment with complete information. Thus, behavior in different treatments can show how different market institutions promote information discovery by traders and induce competitive allocations, such as those described in section 2.1, or instead leave traders stuck in suboptimal Nash allocations described in section 2.2, representing the failure of one or several markets to open. For each treatment we repeat the experiment with the same group for fifteen rounds to facilitate learning. The goods are called "widgets" in the experiment. Buyers and sellers have induced valuations as in the previous section (Table 1), which are constant between rounds and treatments. These payoffs follow the example in Shapley and Shubik (1971), but are scaled by a factor of 20 to form a large discrete space of integer bids and prices. The particular set-up in this table is attractive for the complexity of the problem for the players, which is confirmed by our simulations described in Appendix F.

3.2.1. Double Auction (DA)

The first treatment adapts the double auction commonly used in market experiments. In our version, the game is played in two stages. First, each seller sets a minimum price for her good. This price has to be above her reservation value. When all minimum prices are set, the game proceeds to the trading stage. During the trading stage, buyers are allowed to bid for the goods, and sellers are allowed to reduce their minimum prices. A buyer has a winning bid for a good if her bid is the highest and it is above the current minimum price of the good. To enforce unit demands, buyers who currently hold the winning bid for a good are not permitted to make other bids until some other buyer outbids them.[9] The round ends after 50 seconds of inactivity and buyers with winning bids obtain their corresponding goods. The bidding phase of an average round lasted 3 minutes and 25 seconds. The game continues for fifteen rounds with groups, buyer and seller identities, and reservation values unchanged between rounds, and with sellers revising their minimum prices at the first stage of each round. One round is then chosen at random for payment for each subject. The earnings for buyers are calculated as the difference between the good's value and the bid. Likewise, the earnings for the sellers equal the difference between the bid and the reservation value. Sellers earn zero if they do not sell the good, and no trader risks trading at a loss. Complete experimental instructions are available in the online appendix.

3.2.2. Minimum Price (MP)

The second treatment is set up to give market power to sellers, which should also help coordination. Like in the DA treatment, in the first stage, each seller sets a minimum price for her good. This price has to be above her reservation value, and this is the only way in which sellers actively participate in the market. When all minimum prices are set, the game proceeds to the trading stage with buyers bidding for the goods. This treatment is based on Pérez-Castrillo and Sotomayor (2002, 2017) and relates to job market matching models in Crawford and Knoer (1981) and Kelso, Jr. and Crawford (1982). It also mimics the real estate system that frequently relies on posted prices for houses with a subsequent negotiation phase. As in the sequential game of section 2.2, the commitment ability of sellers should shift the results toward seller-optimal core allocations (upper-right corners in both panels in figure 1).

[9] In our strategic market game it is in principle feasible to allocate multiple non-trivial goods to one buyer, but it can never be an equilibrium outcome. Experimental application in Ott (2009) demonstrates that subjects in a similar environment do not violate this assumption. That is, experimental subjects do not bid when they are the highest bidders for one of the items even if they are allowed to do so.

3.2.3. Pit Trading (PT)

The third treatment is played through open-form bilateral communication between buyers and sellers. Every buyer has three chatboxes for private communication, one for each seller, and similarly, each seller has three chatboxes, one for each buyer. Chatboxes contain two components, one for exchanging messages and one for negotiating the price. Temporary deals are marked in the chatbox shared by a buyer and a seller. Buyers and sellers can simultaneously communicate in the three chatboxes and back out of a deal at any moment, possibly striking another deal. The current negotiated deals are finalized when the round ends after 4 minutes. Traders on the same side of the market can only communicate with the other side of the market and cannot communicate between themselves. No other information about the costs, values, or current negotiation by other players is shown to the subjects. This treatment is motivated by housing markets in which negotiations take place bilaterally over the phone or email. In this scenario, contracts are easier to back out from than in DA or MP.

3.2.4. Pit Trading with Complete Information (CI)

In this treatment all participants could see a table with costs and valuations (similarly to Table 1) during trading. This turns the game into one of complete information and brings the design closer to the theoretical model. This treatment was conducted online from October to December 2020 with the same subject pool (GMU students) and recruitment process but with a higher show-up fee (US$ 10 against US$ 5 in the other treatments). The experimental design for this treatment is otherwise identical to the pit trading treatment.

3.2.5. Robustness of Design

Simplified static complete information models have been historically in use at least since Cournot and Bertrand competition, and are still commonly employed for experiments (for example in advertisement auctions, Edelman and Schwarz, 2010; Fukuda et al., 2013). A faithful dynamic game-theoretic model would require either a shift of the design away from a market experiment or additional assumptions to make strong predictions, for example, a very long experimentation phase for adaptive learning models.[10] Pointing to these problems, Hendon and Tranaes (1991) show that markets as small as 1 seller and 2 buyers can have only non-stationary equilibria and several of them. On the other hand, several results on the convergence of learning dynamics in assignment markets show that static simplification may be warranted. Pradelski and Nax (2019) show that core outcomes are reached by adaptive learning dynamics. The potential function in Demange et al. (1986) can be viewed as evidence for the convergence of best-response dynamics.

[10] Newton and Sawa (2015) quote Boudreau (2012): "Calculating the probability of each stable outcome for a given market under the randomized tâtonnement process is extremely difficult due to the tremendous number of paths that can be involved." This claim has been contested in recent years with Newton and Sawa (2015) in particular offering an analytical solution to the nontransferable utility problem based on stochastic stability, Nax and Pradelski (2015) extending the approach to assignment markets, and Elliott and Nava (2019) offering a solution to the non-cooperative bargaining model in the same markets.

This is a simplification, and there is a vast space of dynamic behavioral strategies that can and are used by subjects. In one of the sessions of the PT treatment, a pair of buyer 3 and seller 2 have decided[11] to take turns offering the whole surplus to each other. They even explained this strategy and convinced another pair to try it. Another group independently came up with the same idea and followed it for one round so that they "each have a high number for at least one of the rounds". According to the chat logs and the submitted bigs, there are exactly 8 occurencies of this across all matched pairs, rounds, and groups. It is not always clear whether the subjects wanted to split the surplus after the experiment, or just were risk-loving. We do not correct for this, although this would strengthen some of our comparisons, because zero payoffs cannot be explained by our bargaining models. Therefore averaging the payoffs or removing the datapoints would improve the performance of PN-Core for the PT and CI treatments.

Another difference between the theoretical part and the experiment is the discrete set of options for bids in the experiment, while the theoretical model has continuous actions. The discrete action space is generally unavoidable without allowing subjects to bid arbitrary irrational numbers or increasingly small fractions. Importantly, however, the discrete finite action space does not affect the set of Nash equilibria. It is easy to show that since the gaps between valuations of different goods are much higher than the smallest bid increment, the equilibrium assignments are unchanged, and equilibrium prices are within the minimum bid increment (1 in this experiment) from the Nash equilibrium.

Unlike discrete bids, the choice of a private information setting for our experiments is non-trivial and has different implications for learning in our treatments. A natural critique would be that due to this private information, in addition to Nash equilibria in the market game, other outcomes may occur because subjects may hold incorrect beliefs about market opportunities. However, the information structure is less relevant for the repeatedly played strategic market game. Since subjects have private values, that is, their payoff only depends on their value/cost and actions of other players (and not the other players' values), the best replies too only require conjectures about the play of opponents. Thus, beliefs about realizations of values/costs are irrelevant as long as observed actions are consistent with the conjectures as required in any self-confirming equilibrium. Formally by Proposition 3 (ii) in Dekel et al. (2004), the set of such self-confirming equilibria reduces to the set of Nash equilibria of the complete information game with the realized payoffs that we study.

Moreover, while in other treatments there is room for bargaining between buyers and sellers, the minimum price treatment MP is similar to multiple auctions in Ott (2009) and Demange et al. (1986). For this reason, even under incomplete information if subjects are only incrementing their bids in small amounts, this reasonable (although not dominant) strategy will lead them to an efficient assignment in a perfect Bayesian

[11] "we have a system [...] her number is 300 mines 420. we switch off between the offer being 300 and 420. so both of us have a 50 percent chance of making 22 dollars plus the 5"

epsilon-equilibrium described in Proposition 3.6 in Ott (2009).

This limited effect of incomplete information may not be true for the bargaining environment of the PT treatment, as bidding is not fully observed and happens in private communication. This is why we conducted the robustness experiment CI with the same subject pool and a similar protocol, but under publicly available cost and value information.

4. Results

We first support our claims with observations about assignments and prices. We start with Table 2 that reports all main results. It can also be compared with Table F.7 of the appendix that reports the same theories for simulated subjects. We then discuss differences in efficiency and seller-buyer surplus division between treatments. Finally, we conduct several checks: stability of outcomes in terms of market dynamics, viability of alternative explanations based on focal points, and stability of bargaining weights. We performed additional tests based on distances between equilibria and experimental price vectors that point in the same direction as the main text. These can be found in Appendix H.

4.1. Descriptive statistics and qualitative results

Predictions can be separated into statements about assignments and statements about prices, and both are reported in Table 2 along with some descriptive statistics. Predictions about prices are generally stronger (as can be seen from simulations in Table F.7 in the appendix).

To summarize, we find support for semi-Walrasian outcomes for DA and MP treatments (Hypothesis 1) both in terms of prices—"Competitive prices for traded goods"—and in terms of assignments—"Nash" assignments. We also find support for the core assignment [*312*] across treatments and limited evidence of core prices (Hypothesis 3), the latter more common in market treatments. We also find fewer semi-Walrasian outcomes in PT and CI treatments with a noticeable shift toward Pareto optimal and PN-Core outcomes (Hypothesis 2).

In more detail, the core assignment [*312*] is the most or the second-most frequent assignment in all treatments as predicted by Hypothesis 3. The second-best Nash assignment [*210*] is more frequent in DA, MP, and CI, whereas [*132*] is more frequent in PT. The third-best assignment [*132*] in our example is the leximin: it delivers the highest possible payoff of 30 to the two worst-off traders. The assignment [*210*] is a semi-Walrasian equilibrium in terms of Mas-Colell (1982) and a Nash equilibrium in terms of Theorem 2.2 with the third market closed. Other Nash assignments are also more frequent in the structured DA and MP treatments. This is consistent with our Hypothesis 1.

Finally, as to be expected from the design, the sellers' share is largest under the MP treatment. The PT treatment is closer to equal split gains. The CI treatment shows surpluses similar to the DA and MP

Table 2: Summary of results across treatments (rounds 6–15)

Assignment (% of observations)	DA	MP	PT	CI
[*312*] (Efficient; Nash)	29	38	59	45.56
[*210*] (Unique 2^{nd} best; Nash)	37	29	6	21.11
[*132*] (3^{rd} best and leximin)	2	4	21	3.33
[*120*] (3^{rd} best)	3	8	5	6.67
[*012*] (3^{rd} best)	8	6	4	8.89
[*310*] (Nash)	9	7	2	4.44
[*010*] (Nash)	1	3	0	2.22
[*102*]	2	2	3	5.56
[*320*] (Nash)	2	1	0	0
[*321*]	1	1	0	1.11
[*130*]	0	1	0	0
[*200*]	4	0	0	1.11
[*032*]	1	0	0	0
[*000*] (Nash)	1	0	0	0
Pareto Optimal (%)	39	55	84	64.44
↪ Pareto Optimal, not dominated by lotteries (%)	29	39	59	45.56
↪ Core = Competitive equilibrium (%)	16	24	10	8.89
Competitive prices for traded goods = 2-blocked (%)	53	49	20	13.33
Pairwise Nash Core for some weights (%)	60	60	84	74.44
Number of violated IC constraints (% of observations):				
1 (%)	21	19	14	25.56
2 (%)	29	28	56	36.67
3 (%)	22	21	17	21.11
≥ 4 (%)	12	8	3	7.78
Efficiency (% of maximum possible total payoff)	87.06	90.19	94.19	90.97
Seller's share (% of surplus)	60.57	64.02	55.53	54.52
Total	100	100	100	90

treatments. Frequent incomplete assignments explain the higher sellers surplus in the MP, DA, and CI treatments since the competition among the three buyers is higher for the two remaining goods.

4.2. Efficiency and sellers' share

The shift in the frequency of assignments between DA/MP and PT treatment in Figure 2 is consistent with Nalbantian and Schotter (1995): the pit trading treatment solves the problem of unmatched players, but makes suboptimal assignments like leximin more probable.

We can see which effect dominates by looking at the overall efficiency. The (incomplete information) PT treatment performs better than the other treatments in terms of efficiency, contrary to what would be expected based on experimental evidence for homogeneous goods. The fourth complete information treatment is close to the DA and MP treatments in terms of efficiency (90.97%), not the PT treatment that tests the same institution.

There is a common belief that auctions outperform decentralized bargaining in terms of the total surplus of traders. Within-study comparisons also suggest this effect, for example Kirchsteiger et al. (2005). Interestingly, our assignment markets and the thin experimental literature on bargaining and matchings with transferable utility (Nalbantian and Schotter, 1995, discussed above) point to the contrary: since renegotiation is now a valuable instrument for efficiency, pit trading treatment outperforms centralized market treatments. Moreover this is achieved, in line with previous bargaining experiments, through higher volume at more equalized prices. Of course this is only suggestive since there is no reasonable way to compare efficiency across studies because of the differences in trading protocols, gains from trade, the number of traders, and the complexity of finding the efficient matching.

We perform both the t-test and the exact permutation Wilcoxon–Mann–Whitney rank-sum test to support the claimed differences in efficiency and sellers' share of the surplus between treatments. The results are shown in Tables 3 and 4. In both cases errors are clustered at the group level, because of the correlation between different periods. Rank-sum test has the advantage of being valid for any distribution, and there is no reason for the efficiency to be normally distributed. The Wilcoxon test is robust to outliers but has the disadvantage of not measuring purely the difference in means, reacting also to differences in shape. Both tests paint a similar picture. The PT treatment's efficiency is significantly higher than the other treatments, while the sellers' surplus share is significantly higher in MP treatment.

Table 3: Ranksum and t-tests for differences in efficiency between treatments

	Dependent variable:	
	WF=100%	WF
MP vs DA	0.090(4.786)	10.000(4.789)
PT vs DA	$-0.300^{***\dagger\dagger\dagger}$(5.714)	$-22.800^{***\dagger\dagger\dagger}$(5.601)
PT vs CI	$-0.134^{**\dagger}$(5.303)	$-10.289^{\dagger\dagger}$(5.261)
MP vs rem. obs.	−0.065(14.735)	−1.745(14.659)
DA vs rem. obs.	$-0.186^{***\dagger\dagger\dagger}$(15.174)	$-15.193^{***\dagger\dagger\dagger}$(15.087)
PT vs rem. obs.	$0.191^{***\dagger\dagger\dagger}$(9.091)	$12.926^{***\dagger\dagger\dagger}$(9.255)
CI vs rem. obs.	0.036(14.880)	1.578(14.909)

Notes: Values in the table are differences in means, Wilcoxon rank-sum statistics in parentheses, *,**,***, †, ††, ††† are Datta and Satten (2005) rank-sum and t-test significance levels for 10%, 5%, 1%. Std. errors clustered at group level using R packages by Robitzsch and Grund (2023), Jiang et al. (2020).

Table 4: Ranksum and t-tests for differences in the sellers' share of the surplus between treatments

	Dependent variable:
	Sellers' share of the surplus
MP vs DA	$0.034^{\dagger\dagger}$(4.697)
PT vs DA	$0.050^{***\dagger\dagger\dagger}$(4.238)
PT vs CI	−0.010(5.106)
MP vs rem. obs.	$0.071^{***\dagger\dagger\dagger}$(13.254)
DA vs rem. obs.	$0.024^{***\dagger}$(13.853)
PT vs rem. obs.	$-0.073^{***\dagger\dagger\dagger}$(11.892)
CI vs rem. obs.	$-0.055^{***\dagger\dagger\dagger}$(16.000)

Notes: Values in the table are differences in means, Wilcoxon rank-sum statistics in parentheses, *,**,***, †, ††, ††† are Datta and Satten (2005) rank-sum and t-test significance levels for 10%, 5%, 1%. Std. errors clustered at group level using R tools by Robitzsch and Grund (2023) and Jiang et al. (2020).

Tables 5 and 6 provide alternative regressions on the frequency of the efficient outcome (WF=100%), total surplus (WF), and sellers' share of the surplus. The results are qualitatively the same, with free-form negotiation determining efficiency, and the sellers' first-move advantage (MP treatment) determining the sellers' surplus share.

Table 5: Determinants of efficiency

	Dependent variable:							
	WF = 100%				WF			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sellers move first (MP)	−0.065			0.090	−1.745			10.000
	(0.078)			(0.068)	(10.762)			(12.644)
Pit trading (PT or CI)		0.218*	0.255**	0.300**		15.469**	17.800**	22.800**
		(0.119)	(0.117)	(0.119)		(7.470)	(8.374)	(9.687)
Complete information (CI)			0.121	0.166			7.511	12.511
			(0.109)	(0.112)			(10.654)	(11.719)
Constant	0.445***	0.372***	0.335***	0.290***	290.345***	285.931***	283.600***	278.600***
	(0.057)	(0.042)	(0.035)	(0.042)	(4.609)	(5.187)	(6.415)	(8.049)
Observations	390	390	390	390	390	390	390	390

Notes: *p<0.1; **p<0.05; ***p<0.01

Robust std. errors clustered at group level in parentheses.

Table 6: Determinants of sellers' share of the surplus

	Dependent variable:				
	Sellers' share of the surplus				
	(1)	(2)	(3)	(4)	(5)
Sellers move first (MP)	0.071***		0.063**		0.034
	(0.023)		(0.025)		(0.025)
Pit trading (PT or CI)		−0.044	−0.022	−0.068**	−0.050
		(0.031)	(0.033)	(0.031)	(0.033)
Complete information (CI)				−0.078**	−0.061*
				(0.032)	(0.035)
Constant	0.569***	0.599***	0.577***	0.623***	0.606***
	(0.015)	(0.015)	(0.018)	(0.013)	(0.019)
Observations	389	389	389	389	389

Notes: *p<0.1; **p<0.05; ***p<0.01

Robust std. errors clustered at group level in parentheses,

one observation with 0 surplus is dropped.

4.3. Market dynamics

The more common outcomes tend to persist from round to round. If we consider every market to be a Markov chain that transitions between different assignments with some unknown probability, then we could statistically compare the observed transitions. Specifically, the empirical probability of renegotiating the efficient assignment again after reaching it in the previous round is 41% for MP, 36% for DA, 85% for PT, and 66% for CI treatment (differences of MP and DA with PT significant at $p < 0.01$ according to Clopper-Pearson confidence intervals, but not with CI). Figure 2 contains four tables with transitions between assignments. The three most frequent assignments discussed above—[*312*], [*210*], and [*132*]—prove to also be persistent with transitions between the first two describing most of the dynamic for the DA, MP, and CI treatments, while the leximin [*132*] and the efficient [*312*] prove to be stable sinks for PT treatment.

The stability of assignments in PT and CI can be attributed to the stability of bargaining weights as an exogenous attribute, which leads to the same allocation across rounds, while Nash equilibria exhibit transient effects of price discovery as subjects move between them. A simple behavioral explanation is that the personal nature of negotiations leads subjects to stick with their partners across rounds.

4.4. Focal points

It is tempting to explain lower efficiency of the CI treatment's and the frequent occurrence of [*210*] by fairness concerns, as subjects observe the surpluses and have a clear focal point of splitting them equally. While this may be the case, it is easy to check that equal weights PN-Core leads to assignment [*120*] instead of [*210*]. In the [*210*] assignment, the second seller, who would obtain 70, and the second buyer, who would obtain 80, both prefer to form a pair for a payoff of 90 each. The equal weight Nash bargaining solution would result in [*312*]. The split is also usually not 50/50 for CI treatment conditional on the assignment [*210*]: it is observed in only 12% of such matched pairs. Therefore, even if knowing the surplus matrix leads to focal points, these are not the equal splits within each pair, and bargaining positions have to reflect the asymmetries of the payoff matrix at least in part.

Overall, in 9% (DA), 6% (MP), 11% (PT), and 17% (CI) of matched pairs, the split between the buyer and seller was exactly 50/50. It is to be expected that this focal point is strong for the complete information treatment. However the numbers obscure the fact that 91% (DA), 100% (MP), 90% (PT), and 71% (CI) of such pairs are buyer 1 – seller 3 and buyer 2 – seller 1. These are the core assignments that allow for a 50/50 split within the core, while the remaining pair buyer 3 – seller 2 has to split the surplus in favor of the seller in any core allocation. The 50/50 split appears to indeed be attractive to subjects, but only when there is no incentive to look for a better match.

More generally, the splits of the surplus within pairs reflect the strength of the players' positions in the market, even if the outcome is not in the core. Consider DA, PT, and CI treatments. Seller 2 tends to get close to a 50% share when matched with buyer 2 (51% and 59%), but a higher share when matched

	000	010	012	032	102	120	130	132	200	210	310	312	320	321
000	0	0	0	0	0	0	0	0	0	1	0	0	0	0
010	0	0	1	0	0	0	0	0	0	0	0	0	0	0
012	0	0	2	0	0	1	0	0	0	4	0	0	0	0
032	0	0	0	0	0	1	0	0	0	0	0	0	0	0
102	0	0	1	0	0	0	0	0	0	0	1	0	0	0
120	0	0	0	0	0	0	0	0	1	1	0	1	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	1	0	1	0	0
200	0	0	0	0	0	0	0	1	0	1	0	1	1	0
210	1	1	1	1	1	0	0	0	0	15	3	11	0	1
310	0	0	0	0	0	0	0	0	0	2	4	1	0	0
312	0	0	2	0	1	1	0	0	1	7	1	10	1	0
320	0	0	0	0	0	0	0	0	0	0	0	2	0	0
321	0	0	0	0	0	0	0	0	0	0	0	1	0	0

(a) Double Auction (DA)

	000	010	012	032	102	120	130	132	200	210	310	312	320	321
000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
010	0	0	0	0	0	0	1	0	0	1	0	0	0	0
012	0	0	0	0	0	1	0	0	0	1	0	4	0	0
032	0	0	0	0	0	0	0	0	0	0	0	0	0	0
102	0	0	0	0	0	0	0	0	0	1	0	1	0	0
120	0	0	1	0	0	0	0	1	0	0	1	4	0	1
130	0	0	0	0	0	0	0	0	0	1	0	0	0	0
132	0	0	0	0	0	1	0	1	0	0	0	1	1	0
200	0	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	3	0	2	0	0	0	0	9	3	10	0	0
310	0	3	0	0	0	2	0	0	0	1	1	0	0	0
312	0	0	2	0	0	2	0	1	0	12	1	14	0	0
320	0	0	0	0	0	0	0	1	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(b) Minimum Price (MP)

	000	010	012	032	102	120	130	132	200	210	310	312	320	321
000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
010	0	0	0	0	0	0	0	0	0	0	0	0	0	0
012	0	0	0	0	0	0	0	1	0	0	0	3	0	0
032	0	0	0	0	0	0	0	0	0	0	0	0	0	0
102	0	0	0	0	1	1	0	0	0	0	0	0	0	0
120	0	0	0	0	0	1	0	2	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	2	1	0	14	0	0	0	3	0	0
200	0	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	0	0	0	1	0	0	0	2	0	1	0	0
310	0	0	1	0	0	0	0	0	0	0	0	1	0	0
312	0	0	3	0	0	1	0	1	0	4	2	44	0	0
320	0	0	0	0	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(c) Pit Trading (PT)

	000	010	012	032	102	120	130	132	200	210	310	312	320	321
000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
010	0	0	0	0	0	0	0	1	0	0	0	0	0	0
012	0	0	1	0	2	0	0	0	0	2	0	2	0	0
032	0	0	0	0	0	0	0	0	0	0	0	0	0	0
102	0	1	1	0	0	0	0	1	1	1	0	0	0	0
120	0	0	0	0	0	1	0	0	0	2	1	1	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	1	2	0	0	0	0	0	0	0	0
200	0	0	1	0	0	0	0	0	0	0	0	0	0	0
210	0	1	3	0	0	2	0	0	0	3	1	7	0	0
310	0	0	0	0	0	1	0	0	0	1	0	1	0	0
312	0	0	0	0	2	0	0	1	0	9	2	23	0	1
320	0	0	0	0	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0	0	1	0	0

(d) Complete Information Pit Trading (CI)

Figure 2: Transitions between assignments (total incidence per treatment)

with other players (63% to 89%). A similar effect is present in MP treatment, but the sellers are already favored by this treatment. It is possible that while players do not always switch partners when a better deal is possible, they still have a "satiation point", a minimum payoff that they consider fair given their other opportunities. The PN-Core develops this idea into a formal solution concept, and its fit to the data is described in the next section.

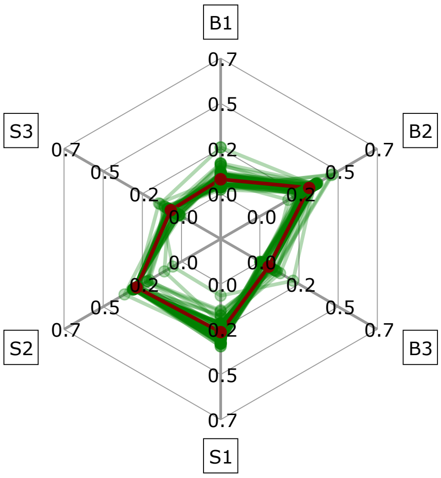
4.5. Empirical Bargaining Weights

We conclude the description of the data with the discussion of PN-Core. The PN-Core requires special attention to avoid overfitting because the theory has low power when weights are free. We can check if significant heterogeneity in weights is necessary to explain the data by minimizing the variance in empirically determined bargaining weights. The weights are at least partially revealed by the data. First, the ratio of the weights within any matched buyer-seller pair is determined by the observed split of the surplus. The incentive compatibility constraints of the PN-Core further restrict the weights across players. Appendix D describes the resulting (mixed-integer) linear program.

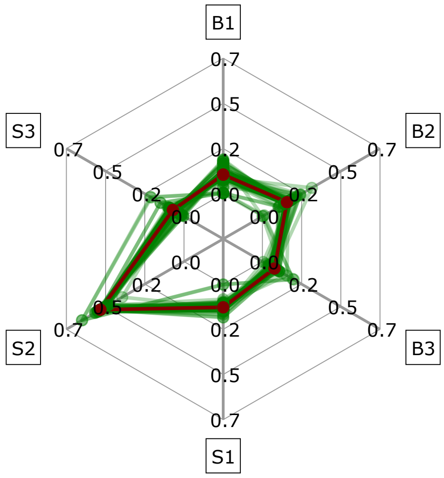
The weights that explain every data point and simultaneously minimize the variance of weights across periods are shown in Figure 3. We only consider here the points consistent with PN-Core for some weights.

For the interpretation of the PN-Core weights, consider, for example, the mean empirical weights of the PT treatment: $(0.15, 0.27, 0.06)$ for the buyers and $(0.17, 0.27, 0.09)$ for the sellers. These weights imply that buyer 2 and seller 2 would obtain half of the surplus a_{22} when matched to each other, but a higher share when matched with other players. Buyer 3 and seller 3 have the least negotiation power and obtain smaller shares. Bargaining with these weights would result in an allocation with assignment [*312*]. Any other allocation is dominated. Consider instead [*213*]. Then buyer 3 and seller 2 can make a better deal: even though their trade surplus a_{23} is smaller than a_{21}, the bargaining weight of buyer 3 is low. Buyer 3 is willing to accept a small share of this smaller surplus, so seller 2 still prefers to switch. Buyer 3 would receive some positive payoff, and seller 2 would get a higher payoff than trading with buyer 1. Public knowledge of buyer 1's fair share decreases her payoff. If only she would accept a lower fraction, the assignment would instead be [*132*], and her payoff would have been higher.

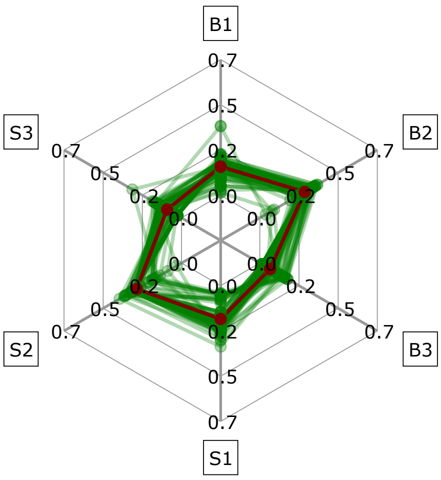
The standard deviations of the weights are 0.1, 0.08, 0.13, 0.09 for DA, MP, PT, and CI. It is perhaps easier to interpret these values in terms of payoffs. The difference in the potential trade surpluses from negotiating within any pair of subjects is within 7%, 18%, 9%, 13% from the mean for DA, MP, PT, and CI. All the variance in prices and assignments among these subjects can therefore be explained by a relatively low variation in what they consider fair. At the same time, the bargaining power has to be skewed in favor of buyer 2, seller 1, and seller 2 to explain the DA treatment, and in favor of seller 2 to explain the MP treatment. The PT and CI treatments are consistent with more uniform weights, further pointing at fairness motives and bargaining outcomes for these treatments.



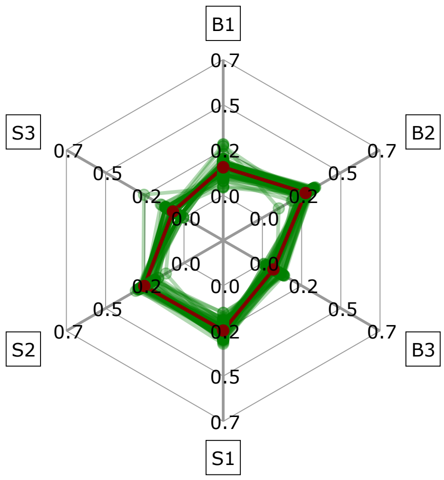
(a) DA



(b) MP



(c) PT



(d) CI

Figure 3: Empirical bargaining weights by treatment

5. Conclusion

The inability of the core concept to capture bargaining and strategic aspects of trading was noted both by Shapley and Shubik (1971) and Von Neumann and Morgenstern (1953), in a critique of the classic Böhm-Bawerk (1891) solution for the horse market assignment problem. As noted by Shapley and Shubik (1971), the choice of the appropriate solution concept is dictated by the institutional form, including the communication structure. This logic is evident in the experiment.

Our experimental results notably follow the 70 years of intuition in experimental markets with negotiation—as first noted by Chamberlin (1948), the volume of trade in such "imperfect" markets is above the equilibrium level, and the prices are below the equilibrium level. Our experiment suggests that these effects extend to heterogeneous goods. We also observe a larger volume of trade and lower seller surplus when comparing the pit trading treatment with the other treatments. The higher volume of trade is consistent with the explanations based on imperfections in the matching procedure, which in our case also helps avoid the mis-coordination of the semi-Walrasian outcomes. The usual explanations for the lower prices are behavioral, the buyer's perception or fairness of seller's prices is also discussed as early as Chamberlin (1948). We complement these explanations with a bargaining model in the heterogeneous good setting that has a similar intuition and fits the data.

Higher efficiency in the treatment with unstructured negotiations could be a suggestive explanation of why open non-centralized negotiations are prevalent in real estate, with the proviso that the opportunities for communication lead to trades at prices that differ from competitive prices, a possibility that indeed was implicitly entertained by both Von Neumann and Morgenstern (1953, p. 564) and Shapley and Shubik (1971, p. 128).

It would be interesting to test the experimental results against the predictions of the strategic bargaining model in Elliott and Nava (2019). Unfortunately, the solution for our 3×3 experimental market appears to be both probabilistic, with multiple assignments occurring with positive probabilities, and difficult to compute.

Finally, the detrimental behavioral effect of complete information evident in the CI treatment could be explained by a fairness motive that leads subjects to disagree more often. It should be noted that, unlike the rest of the analysis, this is an ex-post explanation for an effect that we did not fully anticipate or planned to test.

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Appendix A. Theorem Proofs

Proof of Theorem 2.1. To prove the theorem, consider w such that

$$p_i^S = \begin{cases} p_i' & \text{if } i \in S' \\ \kappa & \text{if } i \notin S' \end{cases}$$

for some $\kappa > \max_{i \in S, j \in B} h_{ij}$, and

$$p_{ij}^B = \begin{cases} p_i' & \text{if } y_{ij}' = 1 \\ 0 & \text{otherwise} \end{cases}.$$

While the assignment $x(y')$ makes zero surplus, every other assignment makes zero or negative surplus, and the assignment $x(y')$ ray-dominates every other zero-surplus assignment. Hence, $F(w) = \{x(y')\}$, as desired.

It is straightforward to check that since prices for goods such that $i \in S'$ are competitive, and prices for goods $i \notin S'$ are prohibitively expensive, no buyer has the incentive to deviate from w to any other bid profile inducing an assignment different from x with positive probability. Similarly, since bids for goods such that $i \in S'$ are competitive, and bids for goods $i \notin S'$ are zero, no seller has the incentive to deviate from w to any other bid profile inducing an assignment different from x with positive probability. □

Proof of Theorem 2.2. Suppose that $F(w) = \{x\}$ and $w = (p_1^B, ..., p_N^B, p_1^S, ..., p_M^S)$ is a Nash equilibrium. First, we claim that the surplus is driven down to zero, i.e.

$$\sum_{i \in S} \sum_{j \in B} x_{ij}(p_{ij}^B - p_i^S) = 0 \text{ for all } x \in F(w).$$

If, on the contrary, for some $j \in B$ and $i \in S$, $x_{ij} = 1$ and $p_{ij}^B > p_i^S$, then buyer j has a profitable deviation to $p_{ij}'^B = p_{ij}^B - \epsilon$ for small enough $\epsilon > 0$ so that x is still preferable for the clearing house after the deviation to any $x' \neq F(w)$. Hence, $x_{ij} = 1$ implies $p_i^S = p_{ij}^B$, and every possible match makes zero or negative surplus.

Now let S' be the subset of sellers whose goods are assigned by x, let y' be the solution to $x(y') = x$, and let $p' = (p_i^S)_{i \in S'}$. We claim that (y', p') is a competitive equilibrium for the economy (S', B) and satisfies (3). Market clearing is guaranteed by the definition of $F(w)$. Profit maximization for each seller i at the given price p_i^S is guaranteed by the fact that since w is a Nash equilibrium, $p_i^S \geq c_i$ so that selling at the price p_i^S is at least as good as not selling. Finally, each buyer $j \in B$ maximizes utility by choosing y_j' given prices p'. For suppose not; then one of the following cases must hold:

(*i*) $x_{ij} = 1$ for some $i \in S$ but there is $i' \in S$ such that $h_{i'j} - p_{i'}^S > h_{ij} - p_i^S$,

(*ii*) $x_{ij} = 0$ for all $i \in S$ but there is $i' \in S$ such that $h_{i'j} - p_{i'}^S > 0$,

(*iii*) $x_{ij} = 1$ for some $i \in S$ but $h_{ij} - p_i^S < 0$.

In case (i), buyer j can deviate to $p'^B_{i'j} = p^S_{i'} + \epsilon$ and $p'^B_{i''j} = 0$ for all $i'' \neq i$ for

$$0 < \epsilon < (h_{i'j} - p^S_{i'}) - (h_{ij} - p^S_i).$$

After the deviation, the clearing house should match j and i' since every other match makes zero or negative surplus by the previous step. Similarly, in case (ii), buyer j can deviate to $p'^B_{i'j} = p^S_{i'} + \epsilon$ and $p'^B_{i''j} = 0$ for all $i'' \neq i$ for

$$0 < \epsilon < h_{i'j} - p^S_{i'}.$$

As in the previous case, the clearing house should match j and i' after the deviation. In case (iii), buyer j can benefit by deviating to $p'^B_{i'j} = 0$ for all i', which guarantees a payoff of zero. Hence, in each of the three cases, w cannot be a Nash equilibrium. □

Appendix B. Additional narrow predictions

Both the asymmetric Nash bargaining solution and the PN-core have very wide predictions. We address this by calculating the power and homogeneity of the empirically determined weights, but here we also contrast them to several theories with narrow or point predictions.

The *least core (L-core)* corresponds to a stochastically stable set of a natural learning dynamic in an assignment game studied in Nax and Pradelski (2015), who show this fact by observing that this set is most robust to one-shot deviations. Formally, the *excesses* of players $i \in B$ and $j \in S$ in some outcome with prices p and payoffs u are

$$e^B_j = u^B_j - \max_{i \in S}(a_{ij} - u^S_i), \text{ and}$$
$$e^S_i = u^S_i - \max_{j \in B}(a_{ij} - u^B_j).$$

The minimal excess is then

$$e_{\min} = \min\left(\min_{i: \sum_{j \in B} x_i j > 0} e_i, \min_{j: \sum_{i \in S} x_i j > 0} e_j \right),$$

and the L-core is the set of states that maximizes the minimum excess $e_{\min}$. In our market the L-core is the following set (with constants rounded to 2 digits): $p_1 = [433.33, 446.66], p_2 = 413.33, p_3 = 386.67$. L-core also generalizes the nucleolus.[12] L-core is a very strong prediction, a set of measure zero in our experimental setup. Consistently with our results for Nash equilibria, in the empirical section we report a relaxed version of L-core that is calculated only for the economy consisting of the goods that were traded.

Another focal point for the subjects is the Shapley value (Shapley, 1953), which can also be motivated as a unique fair allocation rule and a totally stable outcome of the link formation game from Myerson (1977)

[12] Nucleolus for the particular market in our experiment coincides with the kernel, see Nax and Pradelski (2015) who use the same example market.

since the assignment game is superadditive. If subjects were to choose with whom to communicate out of their potential trading partners, and picked an allocation that satisfies equity and efficiency conditions from Myerson (1977), they would arrive at the allocation given by the Shapley value. Unfortunately, the standard Shapley-Myerson value defined on the full coalition structure $2^{S \cup B}$ does not fit the assignment market well. The assignment market traders cannot arbitrarily split payoffs between all players, and we would need to allow lotteries. We calculate the Shapley value under the assumption that any groups of players can agree to split the surplus freely between members and then use the closest feasible point as the prediction. This gives the point corresponding approximately to the price vector $(433.33, 393.00, 385.34)$.

Finally, the leximin solution may be an attractive criterion for egalitarian traders (Sen, 1970). Leximin is obtained by maximizing lexicographically the worst-off trader, then the second worst-off, etc. The leximin can be found through a series of consecutive optimization programs and equals the point with the price vector $(410, 360, 410)$.

Appendix C. Nash Bargaining Solutions for the Non-Convex Game

We will illustrate the Nash bargaining extensions with the following assignment market:

$$A = \begin{bmatrix} 10 & \textcircled{2} \\ \textcircled{2} & 1 \end{bmatrix}$$

Consider the leximin payoff vector $u^{lm} = (u_1^S, u_2^S, u_1^B, u_2^B) = (1, 1, 1, 1)$ attainable in the circled assignment—the worst-off agent cannot obtain a payoff larger than 1 in this or any other assignment. The leximin solution is, by definition, Pareto optimal; however, it does not have to be an asymmetric Nash bargaining solution or be in the core. In this case it is not in the core, because it does not maximize surplus, and it cannot be supported by any asymmetric Nash bargaining solution, which follows from the first order conditions. First, note that the value of the Nash product is 1 and that the weights of the matched buyers and sellers should be the same since the surplus is divided equally. Combining this with the normalization $\sum \alpha = 1$, only one degree of freedom remains. For there to be a Nash bargaining solution, the Nash product value in the other assignment should be less than 1. By solving the first order conditions for 4 in terms of α_2, the weight of the second seller, one finds that the Nash product equals $6.325(0.5 - \alpha_2)^{1-2\alpha_2}\alpha_2^{2\alpha_2}$, the expression that is no less than 1.5 for $0 < \alpha_2 < 0.5$. Thus, there are no Nash bargaining weights supporting u^{lm}.

The Figure C.4 illustrates the buyers' payoffs for this market with a projection of the feasible set into a plane where the payoffs of the first seller and the first buyer are the same, and the payoffs of the second seller and the second buyer are the same. The red and green regions correspond to the feasible allocations in this plane depending on the chosen assignment. The union of these regions is clearly non-convex.

The thick line, the convex hull, is part of the (strongly) Pareto optimal set under lotteries.

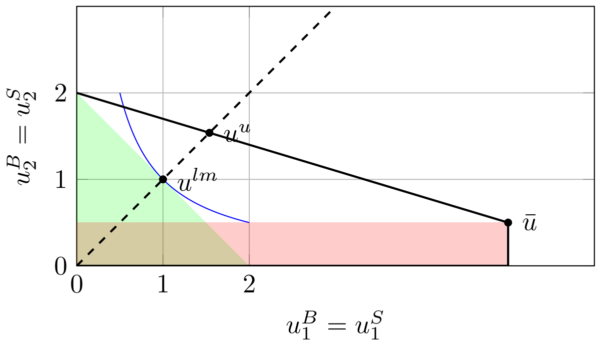


Figure C.4: The three Nash bargaining solutions

There are at least three approaches to dealing with Nash bargaining for such non-convex problems. The difference between them lies in the non-convex regions with imputations that are Pareto dominated by lotteries such as u^{lm}. Whether these imputations are reasonable outcomes depends on the chosen assumptions. The most straightforward solution is to add lotteries directly to the bargaining set, which may not be satisfactory if agents do not have expected utility preferences. Then for some weights of the asymmetric solution the point $u^u = (\frac{20}{13}, \frac{20}{13}, \frac{20}{13}, \frac{20}{13})$ that dominates u^{lm} becomes the Nash bargaining prediction even thought it suggests a probability mixture of the two assignments.

Another approach is a direct extension of the Nash bargaining solution similar to Kaneko (1980) and Herrero (1989). The approach is to fall back to a set-valued solution concept that mechanically returns all maximizers to the weighted Nash function. Clearly all such points will be feasible, but there are some Pareto efficient points that cannot be supported by this concept, such as u^{lm}. With this approach some other nearby point or points would be chosen instead, e.g. with equal weights, the imputation $\bar{u} = \{5, 5, \frac{1}{2}, \frac{1}{2}\}$ is the solution. This approach is referred to as the Nash bargaining solution in this paper and corresponds to the NBS set in Figure E.5.

Finally, the axiomatic approach by Conley and Wilkie (1996) suggests using the unique weakly Pareto optimal point on the ray (shown as a dashed line) from the disagreement point to the Nash bargaining solution on a convex hull of the feasible set(called the utopia point u^u). This set of asymmetric Nash bargaining solutions will span the whole (strongly) Pareto optimal set as one shifts the utopia point. This statement follows from Proposition 2 in Miyakawa (2008) and from the fact that a ray from the disagreement point uniquely maps a point on the convex hull to any point on the weak Pareto frontier of the feasible set. In the same example, the weights $\{3/26, 5/13, 3/26, 5/13\}$ will lead to u^u in the problem for lotteries, which in turn is on a ray with u^{lm}.

There is a reverse problem with this approach, however. In addition to the Pareto set, the set of solutions may include points that are only weakly Pareto optimal even when weights are strictly positive. To avoid these problems, the multi-objective optimization literature suggests checking every resulting solution for strong Pareto optimality and discarding the rest.

The choice between the three approaches depends on whether the outcomes like u^{lm} are deemed less likely because of the necessary sacrifice of efficiency for fairness and some choice rule or alternation between extreme but unfair outcomes like $\bar{u}$ is likely to happen instead. If yes, we need only consider NBS, if not - the whole set PO. All theories are extremely weak spanning most or the whole Pareto frontier.

The structure of the feasible and Pareto sets in Figure C.4 also shows how most of the non-core outcomes are Pareto dominated by lotteries (on the black convex hull) despite being Pareto-optimal in the actual feasible set without lotteries (the red and green regions). The presence of non-core allocations in the data suggests that subjects do not consider lotteries.

Appendix D. Procedure for Determining Empirical Bargaining Weights

We find the weights using the following mixed-integer linear program:

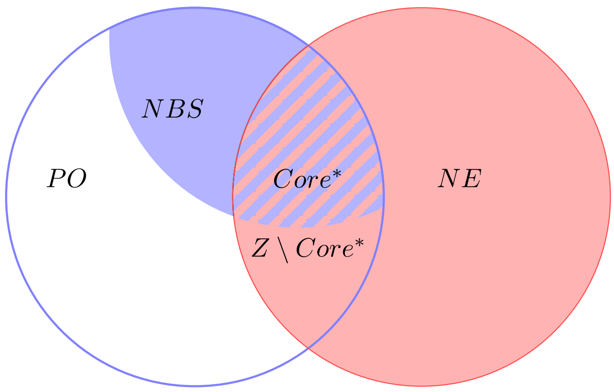
$$\begin{aligned}
&u_j^B(t) * (\alpha_i(t) + \alpha_j(t)) = \alpha_j(t)A(i,j) && \text{for all } j \in B, t \in T \\
&u_i^S(t) * (\alpha_i(t) + \alpha_j(t)) = \alpha_i(t)A(i,j) && \text{for all } i \in S, t \in T \\
&u_j^B(t) * (\alpha_i(t) + \alpha_j(t)) + x(t,i,j)\bar{M} \geq \alpha_j A(i,j) && \text{for all } j \in B, t \in T \\
&u_i^S(t) * (\alpha_i(t) + \alpha_j(t)) + (1 - x(t,i,j))\bar{M} \geq \alpha_i A(i,j) && \text{for all } i \in S, t \in T \\
&\textstyle\sum_{i \in S} \alpha_i(t) + \sum_{j \in B} \alpha_j(t) = 1 && \text{for all } t \in T \\
&\alpha_i(t), \alpha_j(t) \in [0,1] && \text{for all } j \in B, i \in S, t \in T \\
&x(t,i,j) \in \{0,1\} && \text{for all } j \in B, i \in S, t \in T
\end{aligned} \tag{D.1}$$

where the $\bar{M}$ is the sufficiently large "big-M" constant, T is the set of all trading periods or individual observations, $u_j^B(t), u_i^S(t)$ are the experimental payoffs for buyer j and seller i in observation t, $x(t,i,j)$ is an integer constant indicating that the match between i and j in period t was not possible because the buyer (seller) had the better offer or both. If the value of the variable is 0 (1), then the corresponding core constraint for the seller (buyer) does not bind.

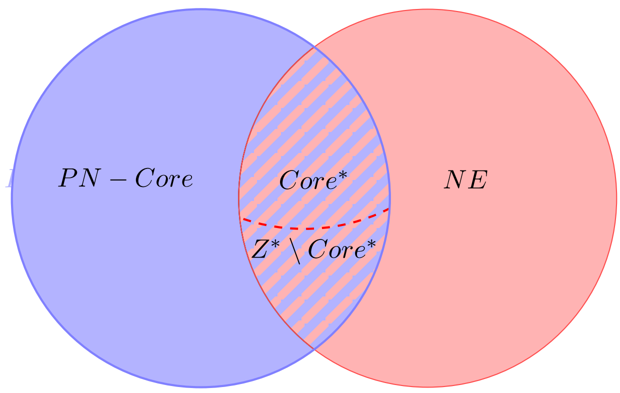
There will usually be a region of solutions. When combined with an objective function that minimizes the variance in α and β across periods, one can estimate the required variance in bargaining power to explain experimental outcomes.

Appendix E. Relationships between solutions

While the Nash equilibria correspond exactly to the semi-Walrasian equilibria in any assignment market, the relationship between the Nash bargaining solution, the core, and Pareto optima is less straightforward and is summarized in Figure E.5. The sets NE, $\mathrm{PN}-Core$ and NBS denote the sets of imputations that can be supported respectively by some Nash equilibrium, $\mathrm{PN}-Core$, and Nash bargaining solution with some weights $\alpha \gg 0$. Any core imputation can be supported by the asymmetric Nash bargaining solution, but not all Pareto optima generally have this property. In fact, the Pareto set is not convex, unless the problem is trivial, and all Pareto optima are attainable in the same assignment.



(a) $Z = NE \cap PO$, $NBS \cap NE = Core^*$.



(b) $Z^* = NE \cap PN - Core$

Figure E.5: Relationship between the Pareto Optima (PO), asymmetric Nash Bargaining Solution (NBS), Pairwise Nash Core (PN-Core), Core, and Nash equilibria (NE)

Figure E.5 shows the inclusions in terms of payoff vectors between the main solution concepts described in this section. The additional set Z in this diagram is the subset of Nash equilibria where for any unmatched buyer and seller $i \in S$ and $j \in B$, we have $a_{ij} = 0$:

$$Z = \{u \in NE, \text{ and for any } (i,j) \in S \times B, m(i) = \emptyset \text{ and } m(j) = \emptyset \implies a_{ij} = 0\}.$$

The sets with a * correspond to the "internal" allocations in the core and Z:

$$Core^* = \{u \in Core, u \gg 0\} \quad \text{and}$$

$$Z^* = \{u \in Z, u_k > 0 \text{ or } m(k) = \emptyset \text{ for all } k \in S \cup B\}.$$

These sets exclude the extreme allocations where some matched players get zero payoffs, which cannot be supported by a Nash bargaining solution with positive weights. The $Core^*$ set can also be empty if some players are always unmatched in the core, for example if there are more sellers than buyers.

The diagram shows that there are two distinct ways in which subjects may "fail" to arrive at the core. One, captured by the Nash equilibrium allows for some players to remain unmatched due to mis-coordination, but ensures that the matched players arrive at competitive prices for the remaining markets. The PN-Core and the other bargaining concepts do not generally allow for unsold goods but distort the equilibrium allocations through a shift in the bargaining power. These sets are disjoint, except for essentially the core region. The rest of the paper argues that either one of these two effects may be a good description of subject behavior, depending on the trading institution.

Below we sketch the proofs for the inclusions in Figure E.5.

The first claim, $NBS \subseteq PO$ follows from homogeneity of degree 1 of the objective function in 4. The objective function, therefore, has a lower value for any Pareto-dominated allocation.

Claim B.1. *$Z = NE \cap PO$.*

Proof. For the allocations without unmatched sellers and buyers, the core is $NE \cap PO$ by definition of the core and $Core \subseteq Z$ by construction of Z. For the allocations with unmatched sellers and buyers, in any Pareto optimal outcome any pair of unmatcheded seller i and buyer j can only have $a_{ij} = 0$, otherwise the allocation is Pareto dominated by the similar one where this pair matches. Conversely, take any payoff vector $u \in Z$. Suppose to contradiction that there is a vector $\hat{u}$ that Pareto dominates u. Since u is a NE, by Theorem 2.2 it is a competitive equilibrium for traded goods and, therefore, Pareto optimal. This implies that for $\hat{u}$ has to be implemented by matching some of the unmatched sellers from the assignment corresponding to u. By definition of Z then, one of the sellers who were matched has to be rematched to a new buyer, and both should receive strictly positive payoffs. But this constitutes a deviation from the Nash equilibrium, contradiction. Therefore, u is Pareto Optimal. Combining the two directions, $Z = NE \cap PO$. □

The next theorem proves that $Core^* \subseteq NBS$.

Theorem B.1. *Every feasible efficient imputation (and thus also every core imputation) with strictly positive payoffs is an asymmetric Nash bargaining solution (NBS) for some vector of weights $\alpha \gg 0$.*

Proof. Suppose $\hat{u} \in Core^*$, attainable at some core assignment x. Let $\hat{U}$ be the maximized surplus obtained in the core. Consider the asymmetric Nash bargaining problem of sharing $\hat{U}$ between all agents with weight vector a, $a_i = \frac{\hat{u}_i^S}{\hat{U}}, a_j = \frac{\hat{u}_j^B}{\hat{U}}$, for $i \in S$, $j \in B$, disregarding the roles and market structure. The Nash bargaining solution is obtained through the following convex optimization problem over a compact set:

$$\max_{u:\sum_{i\in S} u_i^S + \sum_{j\in B} u_j^B \leq \hat{U}} \prod_{i\in S} (u_i^S)^{\alpha_i} \prod_{j\in B} (u_j^B)^{\alpha_j}, \tag{E.1}$$

Solutions to this problem are well-defined and obtained from first order conditions: $u_i^S = a_i\hat{U} = \hat{u}_i^S$ and $u_j^B = a_j\hat{U} = \hat{u}_j^B$ for all $i \in S$ and $j \in B$. Since $U \subseteq \{u : \sum_{i\in S} u_i^S + \sum_{j\in B} u_j^B \leq \hat{U}\}$, the imputation $\hat{u}$ also maximizes the Nash product over a smaller set U. Therefore $\hat{u}$ is the asymmetric Nash bargaining solution for weights a. □

Claim B.2. $NBS \cap NE = Core^*$.

Proof. By Theorem 2.1 $Core \subseteq NE$ and by Theorem B.1. above $Core^* \subseteq NE \cap NBS$. For the converse notice that the value of the objective in (4) is 0 if there are any unmatched players or players with 0 payoffs. Thus in all imputations in NBS all players are matched. The only Nash equilibria are the Core imputations by theorem 2.2. Excluding the imputations with zero payoffs, which are not in NBS, we obtain the set $Core^*$. Thus $NBS \cap NE \subset Core^*$. □

Theorem B.1 does not extend to other Pareto optima that do not maximize the total surplus. An example showing that in general $NBS \neq PO$ is described in Appendix C.

Claim B.3. $Z^* = NE \cap$ *PN-Core.*

Proof. Let $u \notin Z^*$, $u \in NE$. Then two cases are possible. First, suppose $u \notin Z$. Then there are unmatched seller $i \in S$ and buyer $j \in B$ with $a_{ij} > 0$ then i can profitably match with j for strictly positive payoffs in $v_\alpha^{\text{PN}}(ij)$. In this case, the imputation is dominated and $u \notin$ PN-Core. Second, if $u \in Z$, but at least one matched player receives a zero payoff, then this payoff is not an NBS payoff, which is condition (ii) of PN-core. Therefore u is not in PN-Core, and it must be that PN-Core $\cap NE \subseteq Z^*$.

For the converse $Z^* \subseteq NE$ by construction. It remains to show that $Z^* \subseteq$ PN-Core. Take any $u \in Z^*$ and any assignment that makes u feasible. Let k be the number of unsold goods and $\bar{M} = \sum_{k\in S\cup B, m(k)\neq\emptyset} u_k + \sum_{i\in S, m(i)=\emptyset} \max_{\underline{j}}(a_{i\underline{j}} - u_{\underline{j}}^B) + |j \in B, m(j) = \emptyset|$. Take weights $\alpha \gg 0$ such that $\alpha_i = \frac{1}{\bar{M}} u_i^S$ and $\alpha_j = \frac{1}{\bar{M}} u_j^B$ for any matched pair $(i,j) \in S \times B$ and $\alpha_{i'} = \frac{1}{\bar{M}} \max_{\underline{j}}(a_{i\underline{j}} - u_{\underline{j}}^B)$ for all non-traded goods $i' \in S$ and $\alpha_{j'} = 1$ for any unmatched $j' \in B$. By choice of $\bar{M}$ the sum of weights is 1, and $\alpha_i/\alpha_j = u_i^S/u_j^B$ for any matched pair (i,j). The players split the surplus according to their weights, which is condition (ii) of PN-core. Suppose still that $u \notin$ PN-Core. Then for some $(i,j) \in S \times B$ and $\hat{u} \in v_\alpha^{\text{PN}}(i,j)$, we have $\hat{u}_i^S > u_i^S \geq 0$ and $\hat{u}_j^B > u_j^B \geq 0$. But $\hat{u}_j^B + \hat{u}_i^S = a_{ij}$, so $u_i^S + u_j^B < a_{ij}$ and i can profitably match with j. At the same time, $a_{ij} > 0$ since payoffs are non-negative. Then, since $Z^* \subseteq Z$, by definition of Z either i or j has to be matched. If i is matched then u cannot be implemented as a competitive equilibrium for the traded goods and is therefore not a NE imputation by Theorem 2.1. But $Z^* \subseteq NE$ and therefore $u \notin Z^*$. Suppose i is unmatched. The Nash bargaining payoff for j in $v_\alpha^{\text{PN}}(i,j)$ has to be higher than $u_j^B > 0$. However, by choice of weights α_i, α_j, this payoff is $\frac{\alpha_j}{\alpha_i+\alpha_j} a_{ij} = \frac{u_j^B}{a_{i\underline{j}} - u_{\underline{j}}^B + u_j^B} a_{ij} \leq \frac{u_j^B}{a_{ij} - u_j^B + u_j^B} a_{ij} = u_j^B$. By contradiction then, $u \in$ PN-Core.

□

The proof shows that it is possible to explain unsold goods by claiming that the sellers demand a very high proportion of surplus, i.e. their bargaining weights are too high.

Appendix F. Simulated subjects

Appendix F.1. Simulations

The simulations serve two goals. The first is to adjust for the power of Nash equilibria and the other models. The core is a relatively narrow prediction, while Pareto Optimality covers a large part of the feasible set, so a simple horse race between the theories would not give correct results. The second reason is to compare the probability of converging to the prediction depending on the sophistication of the agents. We emulate three strategies. Uniform agents may pick any better reply to current bids and prices with equal probability. The "highest margin" agents also pick a better-reply bid uniformly at random but always for a good with the highest value and price difference. Finally, the "myopic best-reply" agents choose the best reply to current bids and prices, which is a minimum-increment bid for a good with the highest margin.

We emulate market dynamics by having subjects make boundedly rational decisions in random order under all of the experiment's restrictions: not losing money, only increasing bids for buyers or reducing prices for sellers, and the highest bidders forbidden to bid for other goods. Because of these restrictions, this process may fall short of equilibrium or lead to miscoordination. In particular, any complete assignment that does not result in a negative utility is possible.

The types of simulated dynamics are motivated by better and best-reply dynamics in the literature on potentials and weak acyclicity. Other types of dynamics have been shown to converge for such markets including "completely uncoupled" dynamics that do not require knowledge about the structure of the game (Nax and Pradelski, 2015). In our case, where we frequently observe imputations outside of the core region, we are more interested in simple learning dynamics that allow subjects to consistently miss the core rather than the ones that lead only to core refinements. Since we focus on a strategic market game instead of the cooperative game formulation and the core, dynamics defined in terms of better and best replies are also a natural choice.

As with experimental data in Table 2, predictions are separated into statements about assignments and statements about prices. The predictions about prices are generally stronger—the probability of efficient outcome [*312*] is higher than the probability of competitive equilibrium for all (core) or traded goods (Nash equilibria). The probability of arriving at the efficient allocation with boundedly rational decisions is only $20.76 - 29.70\%$. It is much higher only if buyers behave near-optimally with myopic best replies (82.15%). Arriving at competitive equilibrium prices by randomizing is very unlikely, except under myopic best replies. We use arrows to indicate nested theories, e.g., for any imputation $Core \implies PO$.

Table F.7: Simulations: auction with different bidding strategies

Assignment (% of observations)	Uniform	Highest margin	(Myopic) best reply
[*312*] (Efficient)	20.76	29.70	82.15
[*210*] (Unique 2^{nd} best)	19.17	24.22	2.01
[*120*] (3^{rd} best)	19.21	21.72	3.91
[*321*]	15.47	12.38	3.44
[*132*] (3^{rd} best and leximin)	13.74	6.88	7.04
[*231*]	11.65	5.10	1.44
Pareto Optimal (%)	94.56	96.10	99.63
↪ PO, not dominated by lotteries (%)	23.68	32.35	82.39
↪ Core = CE (%)	0.09	0.95	41.53
Competitive prices for traded goods = 2-blocked (%)	20.38	18.41	55.89
Pairwise Nash Core for some weights (%)	86.93	87.69	98.85
Number of violated IC constraints (% of observations):			
1 (%)	1.25	3.57	21.74
2 (%)	24.33	32.03	25.57
3 (%)	70.02	60.91	11.11
≥ 4 (%)	4.31	2.54	0.05
Mean efficiency (% of max total payoff)	88.87	91.32	97.50
Mean seller's share (% of surplus)	79.76	80.11	50.93

Notes: Based on 50000 simulated markets. Players move in random order. Buyers randomly pick a good if they can earn a positive payoff by buying it, sellers gradually reduce the minimum price. Uniform: buyer's new bid is drawn from uniform distribution over all values that give positive payoff. Highest margin: buyers bid for goods that give them the highest payoff at current prices, but choose the new bid uniformly. Myopic best reply: same, but buyers only increment bids by minimum amount. Only unmatched buyers can move. Simulation stops when no further moves are possible.

The high probability of the inefficient assignment [*210*] can be explained by the structure of the market. A trade between buyer 1 and seller 2 cannot happen in a competitive equilibrium, because if $u_2^S \leq 120$ and $u_1^B \geq 20$, then seller 2 would want to sell to buyer 3 instead, and if $u_2^S \geq 100$ and $u_1^B \leq 40$ then buyer 1 would want to buy good 3 instead. Under the rules of the Double Auction and Minimum Price treatments, if current prices indicate the first situation, buyer 3 will be able to outbid buyer 2, and the market would move toward the equilibrium. However, in the second situation, buyer 2 cannot bid for good 3 since this buyer is already the highest bidder for good 2. Thus, the inability to renegotiate a deal may lead to inefficient assignments in the first two treatments.

The simulations can be directly contrasted with experimental behavior in Table 2. While competitive prices are rare under simulated behavior, they occur about a quarter of the time in the experiments. As opposed to random behavior, and consistent with the Nash equilibrium, the frequency of the optimal assignment is related to the frequency of competitive prices and also consistently higher in DA and MP.

Appendix F.2. Predictive Success

The random simulations that we conducted also allow us to calculate the Predictive Success Index or PSI (Selten, 1991), a measure that adjusts the frequency of successful predictions by the power of the test. The values of PSI for the four treatments can be obtained by subtracting values for simulated power tests in Table F.7 from actual results for human subjects in Table 2. Depending on which of the columns in Table F.7 we choose for this, the interpretation of the PSI differs. The uniform simulations adjust for the approximate area of the region consistent with a given theory. For example, the Pareto optimal region is much larger than the core, which makes it easy to arrive at Pareto optimal outcomes, but it does not necessarily make it a better theory.

If instead we use "myopic best replies" we anticipate sophisticated subjects and consequently downplay the predictive power of most of the theories—it is easy to reach the core if one adapts this near-optimal behavior. In this case the PSI captures how human behavior differs from the adaptive process of "myopic best replies" and not necessarily for the better. In other words this PSI indicates by how much the actual human behavior, driven by fairness, bounded rationality and similar concerns, deviates from the outcomes that would be reached by almost optimal adaptive play.

Predictive success for all theories, uniform and sophisticated simulated subjects is presented in Table F.8. We start with the uniform case in the top part. The PSI results are in line with our Hypothesis 2 that matches bargaining and competitive models to institutions. Once again, competitive prices yield good predictions for DA and MP treatments, while Pareto Optimality can generally be reached by human subjects in PT and CI. For PT and CI treatments a less restrictive prediction of efficient assignments performs better when power is taken into account. That is, full efficiency is reached more often, but not necessarily through the competitive price mechanism. The table also reveals that the CI treatment is very poorly described by the competitive equilibrium despite showing similar assignments to the treatments MP and DA with the centralized trade mechanisms.

For more sophisticated simulated agents playing "myopic best replies" PSI is negative, which is to be expected—sophisticated simulated subjects reach the core easily in this case, and the interpretation of PSI is rather how much noise does human behavior introduce into this convergence process—the further the subjects' behavior deviates from the theoretical convergent dynamic, the further is the negative PSI from zero. If the subjects were to behave according to our "myopic best-reply" dynamics they should have converged to the core about 31.53 p.p. more often than in the data for PT treatment. Only the competitive prices for traded goods have PSI near zero for DA and MP treatments. At the same time, the tâtonnement process suggested by the myopic best-replies would reach the Nash equilibria about 3-4 times more frequently than experimental subjects in PT and CI treatments (with PSI -35.89,-42.56) suggesting that other concerns like fairness are driving the outcomes.

We can formalize a statistical test for this table using the properties of variance of difference between two binomial proportions. By definition, any PSI is a difference between binomial proportions of subjects and randomly simulated points consistent with a theory. We use the MKInfer R package (Kohl, 2020) to calculate the confidence intervals for PSI in the table.[13] We can almost always reject that PSI is the same between the PT treatment and the auction treatments DA and MP - the confidence intervals only cross for the core. Comparing CI with MP and DA, we can also reject the null for competitive prices for traded goods. Thus all our statistical tests support that the Nash equilibria fit the data for the auction treatments, but not the bargaining treatments.

Appendix G. Literature review

The heterogenous indivisible goods environment offers a larger spectrum of institutional possibilities than classic works on double auctions and pit trading markets, and it remains largely unexplored. The closest experimental studies to ours are Nalbantian and Schotter (1995), Agranov and Elliott (2021), and He et al. (2022). The first two compare an environment with free-form negotiations and a more structured treatment. The experiment by Nalbantian and Schotter (1995) is conducted under incomplete information about trade surpluses, while Agranov and Elliott (2021) reveal the surplus matrix to subjects. Nalbantian and Schotter (1995) compare the performance of an auction with free-form communication in a 3×3 scenario (this experiment also includes a centralized simultaneous mechanism). Agranov and Elliott (2021), instead, compares a structured Rubinstein bargaining treatment with unstructured free-form bargaining. Similarly to our goal, they combine a realistic unstructured experiment with a stylized game-theoretic model,

[13] There is considerable literature comparing different approaches to calculating confidence intervals for a difference of two proportions. See also Altman et al. (2013).

Table F.8: Predictive Success

Uniform				
Theory	DA	MP	PT	CI
Pareto Optimal	-55.56	-39.56	-10.56	-30.12
	$(-64.55, -45.76)$	$(-49.32, -30.17)$	$(-18.98, -4.65)$	$(-40.41, -21.00)$
↪ Pareto Optimal under lotteries	5.32	15.32	35.32	21.87
	$(-2.68, 14.86)$	$(6.33, 25.12)$	$(25.51, 44.46)$	$(11.97, 32.14)$
↪ Efficient assignment	8.24	17.24	38.24	24.80
	$(0.25, 17.79)$	$(8.33, 27.04)$	$(28.43, 47.38)$	$(14.89, 35.06)$
↪ Core = Competitive equilibrium	15.91	23.91	9.91	8.80
	$(10.01, 24.33)$	$(16.60, 33.14)$	$(5.43, 17.35)$	$(4.48, 16.48)$
Competitive prices for traded goods	32.62	28.62	-0.38	-7.05
	$(22.90, 42.11)$	$(19.03, 38.27)$	$(-7.06, 8.51)$	$(-12.60, 1.50)$
Pairwise Nash Core	-26.93	-26.93	-2.93	-12.49
	$(-36.74, -17.87)$	$(-36.74, -17.87)$	$(-11.36, 2.98)$	$(-22.37, -4.60)$
(Myopic) Best reply				
Theory	DA	MP	PT	CI
Pareto Optimal	-60.63	-44.63	-15.63	-35.19
	$(-69.62, -50.84)$	$(-54.39, -35.25)$	$(-24.05, -9.73)$	$(-45.48, -26.08)$
↪ Pareto Optimal under lotteries	-53.39	-43.39	-23.39	-36.83
	$(-61.38, -43.84)$	$(-52.38, -33.59)$	$(-33.19, -14.25)$	$(-46.73, -26.57)$
↪ Efficient assignment	-53.15	-44.15	-23.15	-36.60
	$(-61.15, -43.61)$	$(-53.06, -34.36)$	$(-32.96, -14.02)$	$(-46.50, -26.33)$
↪ Core = Competitive equilibrium	-25.53	-17.53	-31.53	-32.64
	$(-31.45, -17.10)$	$(-24.85, -8.29)$	$(-36.03, -24.08)$	$(-36.98, -24.95)$
Competitive prices for traded goods	-2.89	-6.89	-35.89	-42.56
	$(-12.61, 6.61)$	$(-16.48, 2.77)$	$(-42.57, -27.00)$	$(-48.11, -34.01)$
Pairwise Nash Core	-38.85	-38.85	-14.85	-24.41
	$(-48.65, -29.79)$	$(-48.65, -29.79)$	$(-23.27, -8.95)$	$(-34.29, -16.53)$

Notes: 5% confidence intervals in parentheses.

specifically a Markov Perfect equilibrium. Finally, He et al. (2022) study decentralized bargaining in markets with different numbers of participants: balanced markets with an equal number of buyers and sellers and unbalanced markets. They document unmatched buyer-seller pairs in their experiment but remain mostly agnostic about the reason. We show that this effect is institution-dependent and creates a trade-off between the efficiency of the matchings and the risk of unrealized gains from trade. The discussion of recontracting in Edgeworth and Walrasian *tâtonnement* is relevant to our treatments and can be found in Walker (1973), with some experimental evidence for positive effect of opportunities for recontracting on efficiency (Smith et al., 1982).

We organize the rest of the related experimental studies in terms of efficiency, distinguishing between organized markets akin to a double auction and free-form negotiations, and between studies with heterogeneous and homogeneous goods. In Table G.9 double auctions for homogeneous goods are close to full efficiency, while bargaining mechanisms generally perform poorer. The few studies of the double auction mechanism for heterogenous goods, however, create a mixed picture.

Appendix H. Prices in Experimental Markets

To further test the differences in prices across treatments, we calculate the mean Euclidian distance from observed price distributions to the semi-Walrasian price regions, conditional on the optimal assignment, the leximin assignment, and for all assignments in Table H.10. For the optimal and leximin assignments, these are the distances to the competitive equilibrium prices, since all goods are sold. Observed prices are closer to competitive prices under the DA and MP treatments conditional on reaching the optimal assignment and in general. Prices are also close to Nash equilibrium under complete information. Conditional on the leximin assignment, observed prices are much further away under the PT and CI treatments, suggesting a non-competitive mechanism for arriving at these prices; values conditional on the leximin assignment for the DA and MP treatment are included for completeness though the leximin assignment is rare for these two treatments. Compared to simulations, experimental subjects reach competitive prices easier than suggested by both the simple uniform strategy and the myopic best-reply strategy.

Calculating distances allows us also to see how close the data points are to the Shapley-Myerson value, L-core, and the leximin price vector, the restrictive theories or predictions outside of the feasible set (Table H.11). The L-core and Shapley-Myerson value are calculated for the economy consisting of only the traded goods for any given data point. For the infeasible Shapley-Myerson payoff vector, we first find the closest feasible price vector. Even though the real subjects are closer to these points than the simulated subjects, we can see that the points are further away from all these predictors and are closer to the core region.

We can also test the core refinements among the points that reach the optimal core assignment [*312*]. These distances are reported in Table H.12. The points do not concentrate at the extremes and are closer

Table G.9: Efficiency in the literature on double auctions and pit trading

Reference	Comments/chosen treatments	Size ($S \times B$)	Het.	DA	Eff., %
Grether and Plott 1984	N-NN (telephone negotiations)	13 (4×9)			100
Smith (1982)	Fig.1, 8 experienced subjects	8 (4×4)		•	100
Williams (1980)	oral auction	8 (4×4)		•	100
Plott and Smith (1978)	oral-bid experiments	8 (4×4)		•	99.8
Hizen and Saijo (2002)	average	6		•	99.7
Smith and Williams (1990)	swastika	27 (11×16 or vv.)		•	99.39
Kirchsteiger, Niederle, and Potters (2005)	double auction	12 (6×6)		•	98.81
Smith (1982)	Fig.2 12 inexperienced subjects	12 (6×6)		•	98.16
Smith et al. (1982)	12-player treatment	6×6		•	97.97
Smith and Williams (1990)	box	22 (11×11)		•	97.9
He et al. (2022)	unbalanced markets, wave 2	7 (3×4)	•		97.5
Kachelmeier and Shehata (1992)	average	8 (4×4)		•	97.41
Williams (1980)	computerized auction, avg. of exp. 1-5	8 (4×4)		•	96.16
Ketcham, Smith and Williams (1984)		8 (4×4)		•	95.8
He et al. (2022)	balanced markets, wave 2	6 (3×3)	•		95.75
Holt (1996)	classroom experiment	18 (9×9)			95.4
Smith et al. (1982)	14-player treatment	14 (7×7)		•	95.4
Hong and Plott (1982)	negotiated price markets	33 (22×11)			95.3
Nalbantian and Schotter (1995)	current free-agency system, CFA	6 (3×3)	•		94.8
Agranov et al. (2022)	supermodular complete inf.	6 (3×3)	•		94
Smith et al. (1982)	8-player treatment	8 (4×4)		•	93.85
Kirchsteiger, Niederle, and Potters (2005)	dec. bargaining	12 (6×6)			92.8
Friedman, Ostroy (1995)	DA1, DA2	8(4×4)		•	92.57
Kimbrough and Smyth (2018)	complete information, no switchover	8 (4×4)		•	92.43
Chamberlin (1948)	classroom experiment	62 (31×31)			90.7
Friedman, Ostroy (1995)	CH1,CH2	8(4×4)			89.84
Ours (PT)	for all periods, last 10 periods in par.	6 (3×3)	•		89.67(94.19)
Agranov and Elliott (2021)	game 30, experiment II	4 (on a graph)	•		89.65
Martinelli, Wang, Zheng (2019)		4 (2 and 2)		•	88.9
Nalbantian and Schotter (1995)	Complete inf. English auction, CIEA	6 (3×3)	•	•	88.3
Ours (DA)	for all periods, last 10 periods in par.	6 (3×3)	•	•	87.42 (87.06)
Ours(MP)	for all periods, last 10 periods in par.	6 (3×3)	•	•	86.92 (90.19)
Ours (CI)	for all periods, last 10 periods in par.	6 (3×3)	•		85.32 (90.97)
Agranov et al. (2022)	supermodular incomplete inf.	6 (3×3)	•		84
Agranov et al. (2022)	submodular complete inf.	6 (3×3)	•		73
Agranov et al. (2022)	submodular incomplete inf.	6 (3×3)	•		39

Notes: Efficiency is aggregated across periods and relevant treatments. Only mechanisms similar to double auctions and free-form negotiations are included, e.g. posted prices are excluded. The 'swastika' is the design where the exchange surplus goes to one side of the market at the CE price, unlike the 'box'. A dot in Het. column indicates heterogenous goods. A dot in DA column and red color indicate a double auction. The values for our treatments are for all 15 rounds, the value for rounds 6-15 is in brackets.

Table H.10: Mean distance between observed/simulated data and Nash equilibrium prices.

	Observed data				Simulations		
Assignment	DA	MP	PT	CI	Uniform	Highest margin	(Myopic) BR
Optimal	6.33	4.69	15.98	10.94	47.00	40.21	12.22
	(18.94)	(13.57)	(25.06)	(16.21)	(20.91)	(21.46)	(27.82)
Leximin	3.64	15.35	34.67	30.32	47.12	40.72	78.27
	(0.10)	(24.00)	(20.24)	(14.64)	(21.15)	(21.86)	(21.68)
All	5.20	7.22	19.39	9.69	46.99	40.20	20.56
	(12.56)	(16.40)	(23.84)	(14.32)	(20.96)	(21.47)	(33.36)

Table H.11: Other theories: Mean distances to Shapley-Myerson value, leximin and the core.

	Observed data				Simulations		
distance to	DA	MP	PT	CI	Uniform	Highest margin	(Myopic) BR
Shapley-Myerson	43.17	62.35	34.90	30.84	69.07	65.89	34.84
	(33.54)	(86.72)	(23.22)	(22.79)	(18.23)	(19.05)	(29.93)
Leximin	62.98	83.10	54.40	51.82	90.97	94.42	59.35
	(14.04)	(68.00)	(12.57)	(10.38)	(26.34)	(23.49)	(16.62)
L-core	30.25	27.24	35.05	40.34	57.32	51.31	36.23
	(23.61)	(20.27)	(28.67)	(30.48)	(20.17)	(20.93)	(32.25)

Table H.12: Mean distances to core refinements (conditional on the optimal assignment)

	Observed data				Simulations		
distance to	DA	MP	PT	CI	Uniform	Highest margin	(Myopic) BR
Seller-optimum	29.73	34.36	43.43	36.13	60.22	52.14	52.82
	(19.84)	(13.43)	(22.76)	(17.32)	(23.78)	(23.64)	(23.28)
Buyer-optimum	30.88	24.08	29.82	25.16	70.45	68.27	18.21
	(17.01)	(21.17)	(23.17)	(15.16)	(19.28)	(20.39)	(30.74)
L-core	15.43	15.56	24.22	19.14	57.33	51.32	28.31
	(23.61)	(20.27)	(28.67)	(30.48)	(20.12)	(20.92)	(26.67)

to the middle core allocations in the L-core even for the MP treatment that favors sellers. The distances are also smaller than for simulated subjects. Thus, conditional on optimal assignment, L-core is a good prediction that does not favor either side.

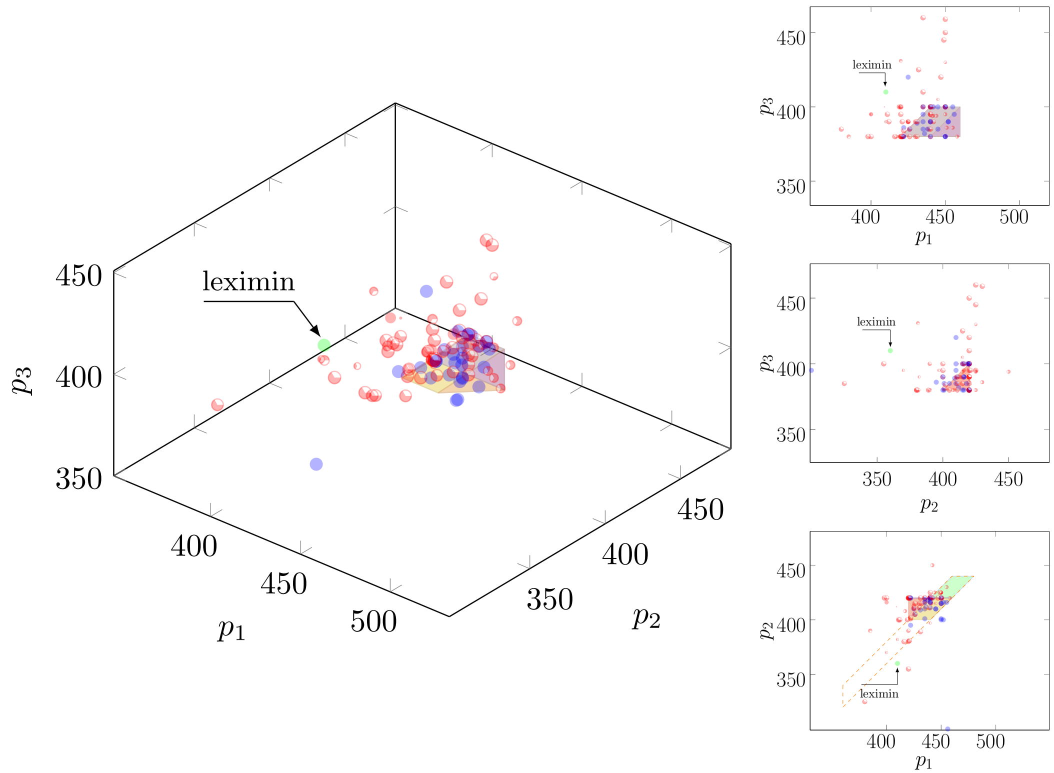
The four panels in Figure H.6 summarize prices for all observations in each of the treatments. The main picture in each of the panels presents the observed price vectors in three dimensions, as well as the core area, shaded in the picture. The smaller pictures present the projection of observed price vectors and the core area in two dimensions, for each pair of markets. The planes limiting the core area in the main picture, as well as the lines limiting the core area in the (projected) smaller pictures, represent various constraints on competitive prices. Intuitively, these constraints make sure that pairs of traders that should not be optimally assigned do not have a profitable opportunity to trade with each other, and that each trader prefers to trade rather than not. Slanted lines in the smaller pictures indicate are related to possible deviations from the core by pairs of traders, while lines that are parallel to either axis are related to participation constraints for the traders. Blue (entire) dots indicate complete assignments, while red dots indicate partial assignments, with the blank space indicating goods that were not traded.

Panel (a) in Figure H.6 illustrates that observed price vectors for complete assignments under the DA treatment are either in or very close to the core area. As the smaller pictures highlight, observed price vectors are often close to or above various constraints. This is clear in particular for the price of good 3. This indicates the resistance of seller 3 to competitive forces pushing down the price of the corresponding good. Intuitively, in the optimal assignment, when the price of good 3 rises too much, buyer 1 would be tempted to deviate and acquire instead good 1 or good 2.

Panel (b) in Figure H.6 illustrates that the observed distribution of price vectors under the MP treatment is similar to that under the DA treatment, with a couple of exceptions. Constraints involving the prices of goods 2 and 3 seem to be binding.

Panel (c) in Figure H.6 illustrates a distribution of price vectors under the PT treatment which differs notably from the other two treatments. There are quite a few observed price vectors for complete assignments which are not close to the core. As illustrated by the smaller pictures, observed prices tend to violate core constraints in two particular dimensions: in several observations, the price of good 3 is too high, and the price of good 2 is too low. This is consistent with some of the observed complete market observations corresponding to the third best assignment [*132*]. In particular, leximin solution prescribes this assignment with the price vector $(410, 360, 410)$, which violates core constraints precisely because the price of good 3 is too high, and the price of good 2 is too low, with respect to competitive prices.

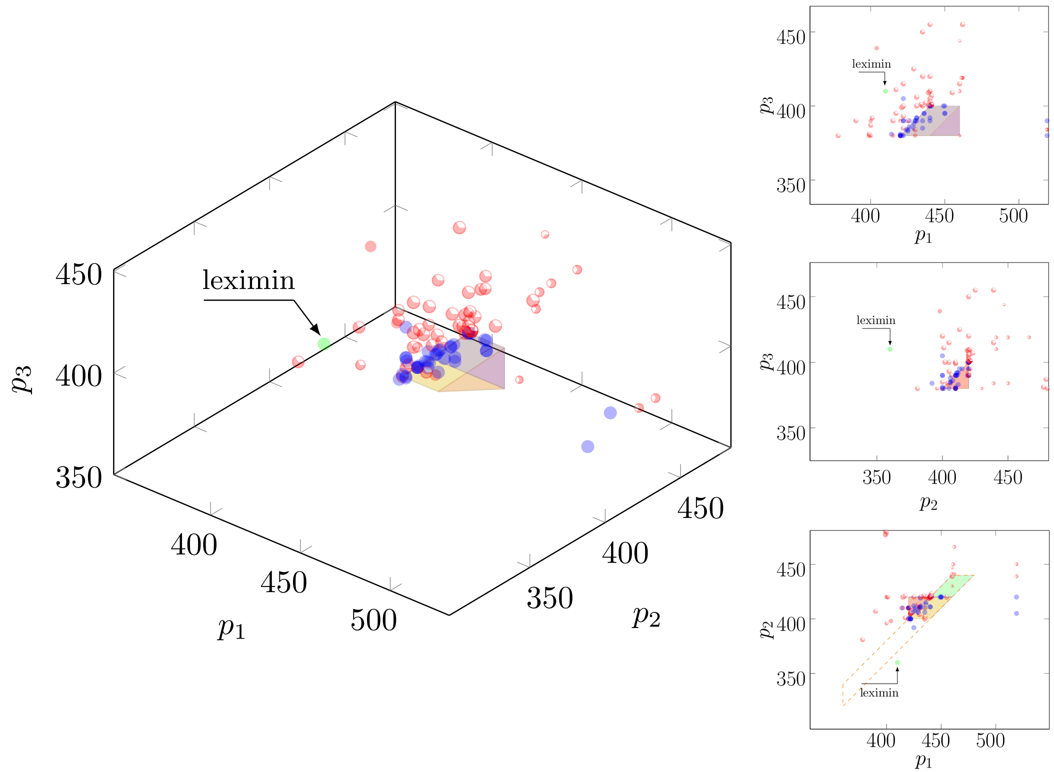
The overall price dynamic is shown in Figure H.7.



(a) Double Auction

Figure H.6: Resulting prices across treatments

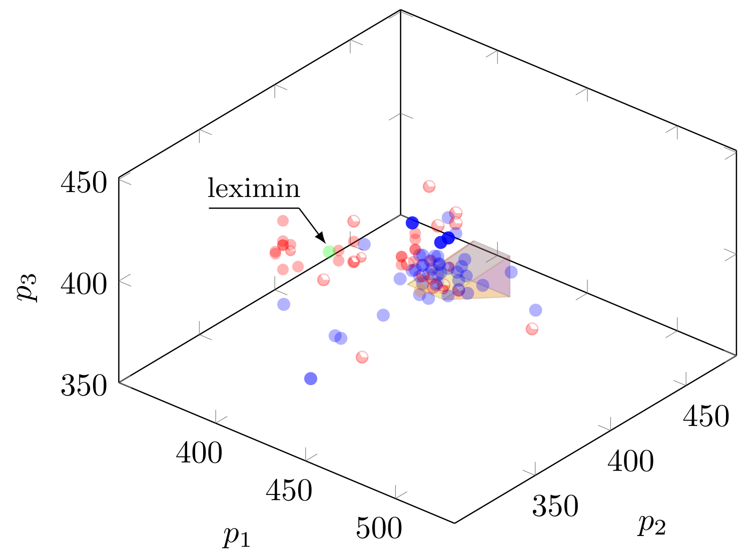
Blue points are rounds with maximized total utility, filling of the markers corresponds to items traded, size of the markers is the total utility. When item is not traded, the minimum price is used instead of the trading price. The green point is the leximin price vector, the green parallelogram in the bottom panel is the core for the game without the third seller, the orange dashed region - the core for the game without the third seller and the third buyer.

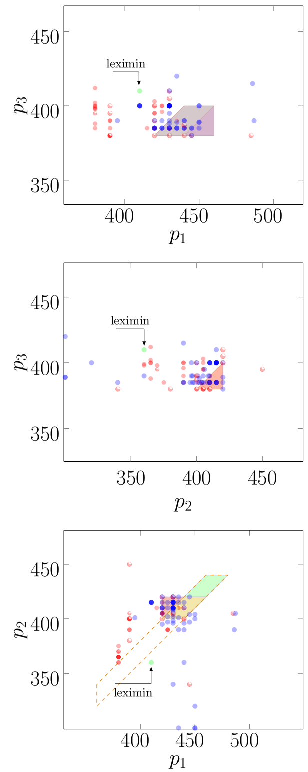


(b) Minimum Price

Figure H.6: Resulting prices across treatments

Blue points are rounds with maximized total utility, filling of the markers corresponds to items traded, size of the markers is the total utility. When item is not traded, the minimum price is used instead of the trading price. The green point is the leximin price vector, the green parallelogram in the bottom panel is the core for the game without the third seller, the orange dashed region - the core for the game without the third seller and the third buyer.

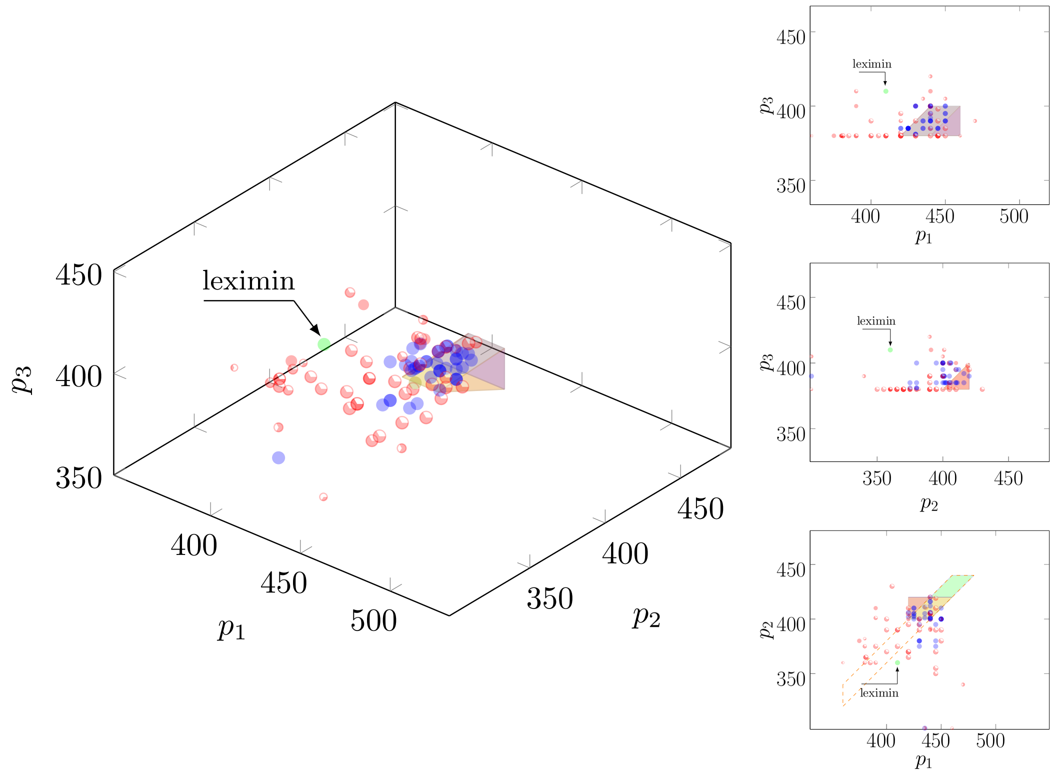




(c) Pit Trading

Figure H.6: Resulting prices across treatments

Blue points are rounds with maximized total utility, filling of the markers corresponds to items traded, size of the markers is the total utility. When item is not traded, the minimum price is used instead of the trading price. The green point is the leximin price vector, the green parallelogram in the bottom panel is the core for the game without the third seller, the orange dashed region - the core for the game without the third seller and the third buyer.



(d) Pit Trading with Complete Information

Figure H.6: Resulting prices across treatments

Blue points are rounds with maximized total utility, filling of the markers corresponds to items traded, size of the markers is the total utility. When item is not traded, the minimum price is used instead of the trading price. The green point is the leximin price vector, the green parallelogram in the bottom panel is the core for the game without the third seller, the orange dashed region - the core for the game without the third seller and the third buyer

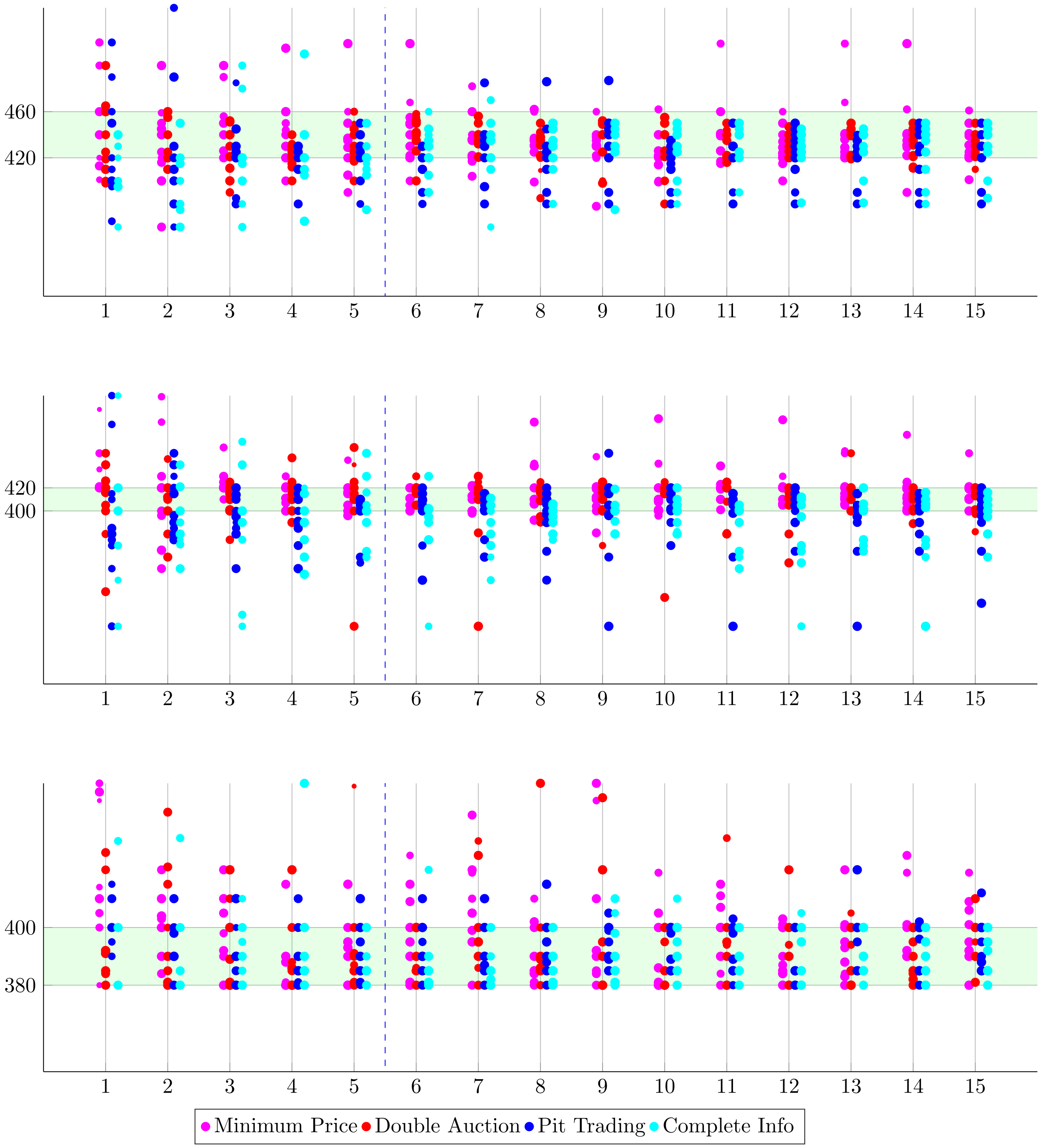


Figure H.7: Resulting prices across rounds

Top – good 1, middle – good 2, bottom – good 3. Light green band is the range of core prices.